CONFIDENTIAL
CLASSIFICATION cancelled
FEB 8 1960
Copy 109
RM L57I18

AERONAUTICS LIBRARY
California Institute of Technology

NACA

RESEARCH MEMORANDUM

ANALOG-COMPUTER INVESTIGATION OF EFFECTS OF FRICTION AND PRELOAD ON THE DYNAMIC LONGITUDINAL CHARACTERISTICS OF A PILOT-AIRPLANE COMBINATION

By Harold L. Crane

Langley Aeronautical Laboratory
Langley Field, Va.

CLASSIFIED DOCUMENT

This material contains information affecting the National Defense of the United States within the meaning of the espionage laws, Title 18, U.S.C., Secs. 793 and 794, the transmission or revelation of which in any manner to an unauthorized person is prohibited by law.

NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

WASHINGTON

November 20, 1957

CLASSIFICATION cancelled

CONFIDENTIAL

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

RESEARCH MEMORANDUM

ANALOG-COMPUTER INVESTIGATION OF EFFECTS OF FRICTION AND PRELOAD ON THE DYNAMIC LONGITUDINAL CHARACTERISTICS OF A PILOT-AIRPLANE COMBINATION

By Harold L. Crane

SUMMARY

With an electric analog computer, an investigation has been made of the effects of control frictions and preloads on the transient longitudinal response of a fighter airplane during abrupt small attitude corrections. The simulation included the airplane dynamics, powered control system, feel system, and a simple linearized pseudopilot. Control frictions at the stick pivot and at the servo valve as well as preloads of the stick and valve were considered individually and in combinations.

It is believed that the results which are presented in the form of time histories and vector diagrams present a more detailed illustration of the effects of stray forces and compensating forces in the longitudinal control system than has previously been available. Consistent with the results of previous studies, the present results show that any of these four friction and preload forces caused some deterioration of the response. However, even a small amount of valve friction caused an oscillatory pitching response during which the phasing of the valve friction was such that it caused energy to be fed into the pitching oscillation of the airplane. Of the other friction and preload forces which were considered, it was found that stick preload was close to 180° out of phase with valve friction and thus could compensate in large measure for valve friction as long as the cycling of the stick encompassed the trim point. Either stick friction or valve preload provided a smaller stabilizing effect primarily through a reduction in the amplitude of the resultant force vector acting on the control system. Some data were obtained on the effects of friction when the damping or inertia of the control system or the pilot lag was varied.

INTRODUCTION

The investigation reported herein was an outgrowth of investigations reported in references 1 to 4 which have been concerned both with the establishment of criteria for desirable control characteristics for powered longitudinal control systems and with the attainment of these desirable characteristics. Reference 1 was based on an analog study of the effects of various types of control feel on the dynamic characteristics of a pilot-airplane combination. References 2 and 3 include discussion of friction effects and of methods for alleviating the friction effects. The two principal sources of control friction which must be considered can be represented as stick pivot friction and friction in the metering valve of the servo actuator. Reference 4 presents the results of an investigation by means of a mechanical simulator of the effects of friction, flexibility, and lost motion in a power control system.

The present paper is closely related to reference 1 in that both were based on analog computer investigations of transient response in pitch to a small step input. In both studies the same closed loop simulation of pilot, power control system, and airplane was used. The analysis of reference 1 has dealt with the adequacy of spring feel with or without compensation for dynamic-pressure variations and of response feel produced by normal acceleration and pitching acceleration as provided, for example, by a pair of bobweights. The purpose of the present investigation was to use the simulation adopted for reference 1 to study effects of friction in a longitudinal control system and to determine how the frictions might affect the adequacy of the spring feel and response feel systems of reference 1.

The analogy of reference 1 was for a closed-loop system which included a simplified pilot simulation based on linear pilot behavior. It is known that, when the need arises, the human pilot almost instinctively achieves closer control by reacting in a nonlinear manner. However, for this series of analog studies it was thought that the restriction to linear pilot behavior had the advantages of producing results that represent a desirably simple mode of control for the human or automatic pilot as well as providing a critical measure of control system characteristics. In addition, when unalterable pilot behavior was provided, the effect of certain feel and control system variations could be more easily detected. The results of this friction investigation, of course, apply more directly to systems which incorporate a force-type autopilot.

SYMBOLS

f	friction, lb
p	preload, lb
C	viscous damping coefficient, ft-lb/radian/sec
K	spring constant, ft-lb/radian
K_θ	pilot gain for response to pitch attitude, lb/radian
$K_{\dot{\theta}}$	pilot gain for response to pitching velocity, lb/radian/sec
K_δ	pilot gain for response to control deflection, lb/radian
K_a	static gearing between stick and elevator, radian/radian
K_b	ratio of valve-arm travel to stick travel with elevator fixed or static gearing between elevator deflection error and valve, radian/radian
K_c	gain between valve deflection and elevator rate, radians/sec/radian
F	pilot applied control force, lb
T	pilot applied control torque, ft-lb
R	resultant driving force - the vector sum of pilot force plus frictions and preloads
M	pitching moment, ft-lb
θ	pitch attitude, radian
δ	deflection, radian
I	moment of inertia of control system, slug-ft^2
p	differential operator, $\frac{d}{dt}$
τ	lag, sec
l	stick length, ft

| | indicates absolute value

T_5 time to damp to within 5 percent of steady-state value, sec

ϕ phase angle, deg

a_n normal acceleration, ft/sec^2

Subscripts:

s stick

v valve

e elevator

ϵ error, difference between input and output

i input or command

o output

p pilot

ss steady state

Dots above symbols indicate differentiations with respect to time.

DISCUSSION OF TEST CONDITIONS

Airplane

The airplane assumed for the present investigation was the same as that used in reference 1. The airplane was typical of fighter size and weight and was equipped with an irreversible hydraulically operated longitudinal control system. The airplane considered had good handling qualities. Throughout this paper the flight operating condition assumed was a speed of 600 feet per second at an altitude of 20,000 feet with a static margin of 5 percent mean aerodynamic chord. For the test condition the airplane undamped period was 2.5 seconds and the damping was 65 percent of critical. A schematic diagram of the longitudinal control system is shown in figure 1. In the block diagram of the simulator shown in figure 2, the airplane longitudinal dynamics are represented by the transfer function $\frac{\dot{\theta}}{\delta_e} = \frac{21.97p + 25.20}{p^2 + 3.37p + 6.69}$.

Feel System

Since only one flight operating condition (at constant dynamic pressure) was to be considered, it was adequate to use simply a centering spring on the stick as had been used for one portion of the control feel investigation of reference 1. The feel system is represented in the left center of the block diagram of figure 2, and the standard system parameters are:

Force per unit normal acceleration, lb/g	4.5
Force per unit elevator deflection, lb/deg	5
Control system inertia, I, slug-ft^2	0.8
Damping at stick, C_s, ft-lb/radian/sec	44.8
Stick spring constant, K_s, ft-lb/radian	625
Stick length, l, ft	2
Natural frequency, radians/sec	28
Damping, percent of critical	100

Servo System

The control system was assumed to be powered by a hydraulic actuator. The hydraulic metering valve was assumed to be connected to the control stick by a rigid link which had no play in the attachment fittings. The valve inertia was assumed to be negligible. The valve was provided with spring centering K_v of 573 foot-pounds per radian and with viscous damping C_v of 100 foot-pounds per radian per second, both values measured at the valve arm. (It had originally been intended that these values of valve centering and damping be normally equal to zero except during specific checks on the effects of valve centering and damping. However, partly through an oversight, and partly because the amounts of valve centering and damping had a very slight effect on airplane response, valve centering and damping were present on most of the test runs.)

In a power control system of this type the valve deflection is approximately in phase with stick rate within the normal range of operating frequencies. As a result the valve spring centering force was fed back to the stick approximately in phase with the viscous damping at the stick and was equivalent to a 2.5 to 25 percent increase in damping at the stick depending on the value of K_b. The amount of damping applied at the valve was the practical equivalent to a 25 to 250 percent increase in control system inertia for K_b values of 0.1 to 1.0.

The hydraulic servomechanism appears at the right center of the block diagram of figure 2. The three gain constants used in simulating the servomechanism were:

K_a static gearing between stick and elevator, radian/radian

K_b ratio of valve arm travel to stick travel with elevator fixed or static gearing between elevator deflection error and valve, radian/radian

K_c elevator deflection rate per unit valve arm deflection, radians/sec/radian

The gain K_a was kept at 1 radian per radian and the gain K_c was kept at 50 radians per second per radian throughout. A range of values of K_b from 0.1 to 1.0 was used which corresponds to lags in the servomechanism of from 0.02 second (at $K_b = 1$) to 0.20 second (at $K_b = 0.1$). However, a value of K_b of 0.4 (for which τ equals 0.05 sec) was adopted as a standard value and was used for most of the runs.

The frequency response of the simulated hydraulic actuator is presented in figure 3 for three values of lag. The transfer functions δ_e/δ_s, δ_v/δ_s, and $\delta_v/\dot{\delta}_s$ are shown for circular frequencies up to 12 radians per second. Oscillatory responses of the complete closed-loop system usually had a frequency of about 3 radians per second which was low enough that control system dynamics were not an important factor in this investigation.

The phase lags in the actuator system increased both in proportion to the frequency and in inverse proportion to K_b. As is shown on figure 3, the lag of either δ_v with respect to $\dot{\delta}_s$ or δ_e with respect to δ_s at $\omega = 3$ and $K_b = 0.4$ was approximately 9°.

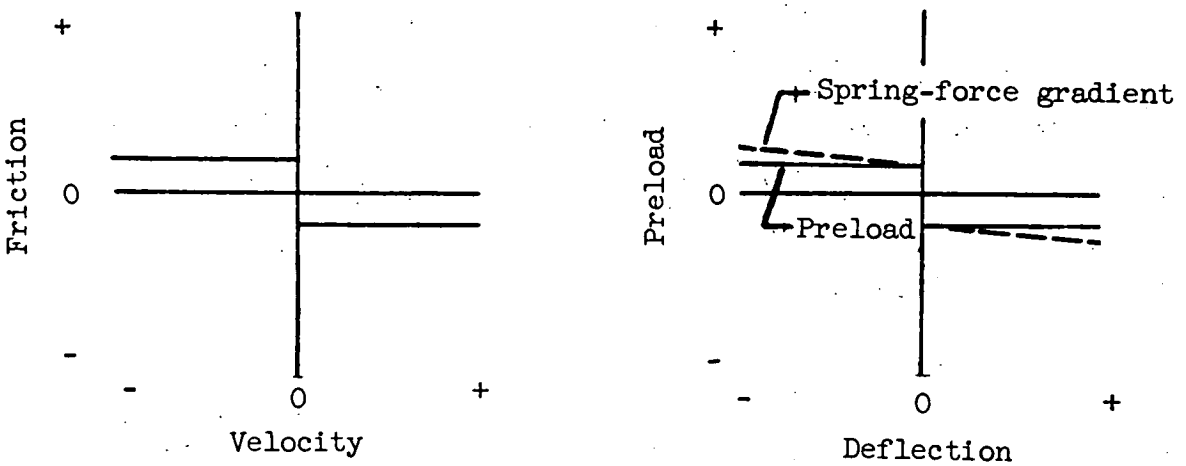
Pilot Simulator

The pilot simulator incorporated two linear lags of 0.15 second each corresponding to the perception lag and reaction lag of the human pilot. Lags and quantities which the pilot simulator sensed were based on the results of reference 1 which assumed that the pseudopilot responded primarily to pitch attitude and which showed that near-optimum airplane response was then obtained with a limited degree of pitch rate and control deflection sensing. (It is known that for many tasks the human pilot responds primarily to pitch attitude.) The optimum gain settings were $K_\theta = 100$, $K_{\dot{\theta}} = 25$, and $K_\delta = 80$ for the ideal case when no stray

forces were considered. These gains have been used as standard values in the present investigation. The θ and $\dot{\theta}$ gains were varied in the present investigation as became necessary to compensate for increasing magnitudes of stray forces, but the ratio of the two gains $K_\theta/K_{\dot{\theta}}$ was kept equal to 4. Large variations in K_δ were found to have a very small effect on system response; thus, K_δ was left unchanged throughout this investigation.

Frictions and Preloads

The effects of stick pivot friction, friction at the metering valve of a hydraulic actuator between the valve spool and the valve cylinder, stick preload, and valve preload on the closed-loop transient longitudinal response of an airplane were investigated. The forces generated by and used in the computer corresponded closely to the following sketches:



This representation of friction is usually called coulomb friction. Coulomb friction probably corresponds to the friction in a control system which is disturbed intentionally or otherwise by a mechanical vibration. No effort has been made in the present investigation to simulate the initial peaking of "stiction" or breakout friction.

The size of the individual frictions and preloads as felt at the stick ranged from a fraction of a pound up to values of several pounds. The higher values either individually or collectively would border on being excessive according to the military flying qualities specifications of reference 5.

A preload force such as that shown in the sketch is normally obtained in combination with a spring-force gradient by the use of preloaded centering springs. This device has been applied frequently to obtain the positive static centering of the control required by reference 5 when friction is present in the control system.

RESULTS AND DISCUSSION

Approximately 500 runs were recorded with the analog computer in which the principal items varied between runs were frictions, preloads, pilot gains, and the mechanical advantage between stick and valve. Some effort was also made to determine the effects of variation in other system constants such as inertia, damping, centering, and pilot lag. About 50 of the more significant runs are presented in the form of time histories and also, when oscillatory response with approximately constant amplitude occurred, as vector diagrams. Tables I to III list the system and pilot gains, the stray forces present, the values of percent initial overshoot, and time for achieving ±5 percent of the steady response for these 50 runs.

The simulated task was usually a step change in attitude of 0.025 radian. A small correction was used because it was thought that this task would tend to provide a critical measure of tolerance to stray force. In the usual case the following quantities were recorded: pitch attitude, stick deflection, pilot force, valve deflection or rate of elevator deflection, stick friction, valve friction, stick preload, and valve preload. In order to keep the traces on scale, ordinate scale changes were frequently made.

In order to obtain an overall picture of the results, the attention of the reader should be directed primarily to the top item on the time histories, the plot of pitch-attitude response. Further insight into the effects of friction and into the quality of the simulation used can be gained from a subsequent more detailed examination of the time histories.

It should be emphasized that the results of the present investigation were obtained with linear pilot response. A human pilot using nonlinear response when nonlinear control feel characteristics were encountered could have achieved closer control. However, the technique used in this paper is considered to provide a critical measure of desirable control characteristics in the presence of nonlinearities introduced by friction.

Response With No Friction or Preload

Figure 4(a) shows the transient response obtained for a pitch-attitude correction of 0.025 radian with standard pilot gains and no friction or preload. (The plots of pitch attitude angle θ are presented in normalized form for which the units are radians $\times$ 40.) It should be noted that in this case the damping was near critical, there was no overshoot, and the response was within 5 percent of the desired value in 5 seconds. This case has been adopted from the investigation of feel systems of reference 1 for use as a standard of comparison for the present investigation. It is considered representative of satisfactory response. Figure 4(b) shows the result of doubling the primary pilot gains K_θ and $K_{\dot{\theta}}$, again with no friction or preload. In this case the response became a lightly damped oscillation.

Vector diagrams have been constructed for cases such as that of figure 4(b) which resulted in oscillatory response. The purpose was to illustrate the effects of friction and preload forces on the phasing of control force, resultant driving force, control deflection, and airplane pitching velocity. The resultant driving force is defined here as the vector sum of pilot control force plus frictions and preloads. Only the orientation of vectors representing control deflection, pitching moment due to control deflection, and pitching velocity have been determined. Figure 5 presents a vector diagram for the no-friction case of figure 4(b), as well as an example with stick friction present which will be discussed in the next section. In figure 5(a) the components of force in the system due to inertia, viscous damping at the stick or servo valve, and centering at the stick or valve are shown along with the pilot control force vector. The more important of the internal forces in this case were the damping and centering at the stick. Within the limited accuracy of the diagram the sum of all the force vectors shown in figure 5(a) is zero.

In addition to the force vectors for the control system, figure 5(a) shows the phase lags between stick deflection, and elevator deflection or the pitching moment due to elevator deflection M_e, and pitching velocity $\dot{\theta}$. Although the pseudopilot was attempting to damp out the pitching oscillation, the phase diagram of figure 5(a) shows that the resulting pitching moment due to control deflection had a large component in phase with the airplane pitching velocity $\dot{\theta}$. The pilot was therefore feeding energy into the pitching oscillation. The amount of energy fed into the motion nearly counteracted the inherent pitch damping of the airplane so that a lightly damped oscillation resulted. (See fig. 4(b).)

Effects of Stick Friction

Figure 6 illustrates the effect of 1/2 pound and of 1 pound of stick pivot friction on the transient pitch-attitude response when a correction of 0.025 radian was required. In some cases including these cases presented in figure 6 the stray force traces were subject to a high-frequency oscillation between the limiting values. In this event the effective friction or preload at any instant was the mean value. This defect was caused by the high gain circuit used in the simulation of friction or preload. Comparison of the data of figure 6 with the no friction result indicates that the presence of stick friction reduced the ability of the pilot to make an accurate small-attitude correction. In this case there was moderate overshoot of the pitch attitude. With linear pilot response an increase in steady-state error is likely to occur because of the presence of stick friction, the possible increase in steady-state error being proportional to the stick friction.

The effect of a given amount of stick friction is dependent on the size of the desired pitch-attitude correction. Figure 7 shows the effect of a 50-percent reduction and of a 300-percent increase of the desired attitude correction with 1/2 pound of stick friction and standard pseudopilot gains. The airplane response was stable enough in either case. However, the steady-state error introduced by the stick friction was negligible for the larger correction but increased to about 30 percent for the smaller 0.0125 radian correction of the attitude angle. Since the initial pilot force output for a correction of 0.0125 radian was about $1\frac{1}{2}$ pounds regardless of the amount of friction, increasing the stick friction to 1 pound (record not shown) increased the steady-state error to over 60 percent. The effect of 1 pound of stick friction was about equivalent to making the pseudopilot insensitive to an attitude error of 0.008 radian or 1/2 degree.

The data of figure 8 show that, where the stick friction (3 pounds) exceeded the maximum pilot force output (2.5 pounds for standard gains $K_\theta = 100$, etc.) no deflection of the controls or airplane response occurred. A 50-percent increase in pseudopilot gain to $K_\theta = 150$, $K_{\dot{\theta}} = 37.5$ resulted in a response which was in error by 70 percent. For $K_\theta = 200$ and $K_{\dot{\theta}} = 50$ the steady-state error was -40 percent. Although neither of these responses would be considered satisfactory, these results indicate that excessive stick friction can be handled by increase in pilot gain without causing the response to become unstable.

With 1 pound of stick friction present, a 50-percent increase K_θ and $K_{\dot{\theta}}$ resulted in an oscillation of approximately constant amplitude (not shown). A vector diagram of this case including an approximated friction

vector is presented in figure 5(b). The internal force components for the control system which were presented in figure 5(a) will not be shown in subsequent vector diagrams. In figure 5(b) the stick friction f_s is seen to cause the resultant driving force R to lag the pilot force F by about 35° and to have about 75 percent the amplitude of F. The phase shift is destabilizing to the complete system and the reduction of amplitude is stabilizing. In this case the destabilizing phase shift proved to be the stronger effect and neutral oscillatory stability occurred at $K_\theta = 150$ and $K_{\dot{\theta}} = 37.5$ compared to occurrence at $K_\theta = 200$ and $K_{\dot{\theta}} = 50$ with no friction.

The foregoing results indicate that the effects of stick pivot friction on the stability of the system are dependent upon the detailed relation between the magnitude of the friction, the magnitude of the attitude correction, and the gain level of the pilot. When these three factors combine so that the friction levels are very small relative to the forces applied by the pilot, the response of the system will obviously approach the response with no friction. For intermediate levels of stick pivot friction the unstable phase shift of the resultant driving force dominates the response and the friction tends to reduce the stability of the system. When these three factors combine so that the friction level approaches the magnitude of the force applied by the pilot, the marked reduction in the resultant driving force increases the system stability but severely interferes with the static accuracy of the system. Thus, if a pilot is capable of adjusting his gain levels so as to apply forces only slightly greater than the friction level he can perform rough corrections by using linear control procedures in the presence of stick pivot friction without destabilizing the system.

Effects of Valve Friction

When even a small amount of valve friction was added to the system, the airplane response became oscillatory. Time histories with the valve friction equal to 1/2 pound and 1 pound are presented in figure 9. A vector diagram for the time history of figure 9(a) is presented in figure 10(a) which shows that valve friction introduced lag (though less than stick friction did) into the resultant driving force R and also increased the amplitude of R compared with the value of force input F which would exist with no friction. Both of these effects are destabilizing. As a result with valve friction present with standard pilot gains, $K_\theta = 100$ and $K_{\dot{\theta}} = 25$, the airplane response included an oscillatory mode of constant amplitude. The oscillation remained neutrally stable when the pilot gains were reduced to $K_\theta = 50$ and $K_{\dot{\theta}} = 12.5$, but the amplitude of oscillation was reduced proportionately.

For cases in which the longitudinal control and the airplane were performing a constant-amplitude oscillation, the effects of valve friction on the oscillating system can be described in another way. It can be said that the valve-friction vector produced an increment of elevator deflection and of pitching moment. This increment of pitching moment was almost in phase with airplane pitching velocity and therefore fed the bulk of its energy into the oscillation in opposition to the damping in pitch of the airplane.

When the desired pitch-attitude correction was reduced 50 percent to 0.0125 radian with 1 pound of valve friction present and standard pilot gains, the response remained oscillatory, and the amplitude of the oscillation increased to 50 percent of the desired attitude change. (See fig. 11.) When the desired attitude correction was 4 times the usual value with 1/2 pound of valve friction present, the amplitude of the oscillation was reduced to 5 percent of the desired correction. However, the actual amplitudes in angular units of the oscillations were proportional to the amount of valve friction in the two cases. Thus the data of figures 9 and 11 indicate that the presence of valve friction caused hunting oscillations to exist under all conditions of valve friction level, magnitude of pitch-attitude correction, and pilot gain level. The only effect of variation in these quantities was to alter the amplitude of the hunting oscillation.

Combined Frictions

Examples of the effects of combined stick and valve friction are shown in the time histories of figure 12. In figure 12(b) with f_s = 1 pound and f_v = 1/2 pound, it was apparent that the stick friction had a stabilizing effect on the oscillation caused by the valve friction. A comparison of the vector diagrams of figures 10(a) and 10(b) illustrates some of the effects of adding stick friction to a system which was oscillatory because of the presence of valve friction. The vector diagram of figure 10(b) indicates that, even though stick friction caused a destabilizing phase shift of the vector R, this effect was more than compensated for by a reduction in the amplitude ratio of R to F which also resulted. The use of stick friction in this manner to help stabilize a power control system when valve friction was present has been previously demonstrated in reference 4.

Effects of Stick Preload

Prior to the investigation of frictions and preloads in combination, records were taken with preload alone in the system. As is shown in figure 13, with stick preload present the airplane pitch-attitude response

became less oscillatory than the no-friction case (fig. 4(a)) and was almost exponential when the preload was increased to 1 pound. However, as a result the response became more sluggish. In addition, preload does interfere somewhat with the precision with which small corrections can be made (with linear pilot behavior) because for small corrections the pilot force application will be within the preload and no control action will result.

Effects of Valve Preload

The time histories of figure 14 show that, with valve preload alone present, the airplane response was almost identical to what it was with only stick friction present. Since valve displacement was approximately in phase with stick velocity, valve preload and stick friction should be approximately in phase with each other and should have nearly the same effect on airplane pitching response. Therefore, as long as there is no play or flexibility between stick and valve, stick friction and valve preload have a similar effect on system response. This correspondence between the effects of stick friction and valve preload is modified by control system lag, flexibility, or lost motion, all of which cause p_V to lag f_S. A discussion of the effects of flexibility and lost motion is contained in reference 4. A discussion of the equivalent effects of stick friction and valve preload is found in reference 2.

Preload as Compensation for Friction

A series of runs such as those presented in figures 15 and 16 were made to investigate the effects on transient pitching response of using preload in combination with friction forces. The stabilizing effect of stick preload, previously discussed when figures 4(a) and 13 were compared, remains evident with either stick friction or valve friction present. Where the system already had adequate stability as was the case with no friction or with 1 pound of stick friction, the added stability was not desirable as the response became more sluggish. (See fig. 15(a).) However, with valve friction present, the stability increment due to stick preload was added to a system which had neutral oscillatory stability. As is evident, particularly from a comparison of figures 16(b) and 9(a), a considerable improvement in response resulted. It should be noted, however, that stabilizing effects of stick preload only occur when the control motion encompasses the trim position. The stabilizing effects of stick preload would be reduced by control system flexibility. A comparison of figures 15(b) and 12(b) shows that in the presence of valve friction the effects of valve preload and stick friction are identical.

Vector diagrams are presented in figure 17 to illustrate the relative effectiveness of stick preload and valve preload for stabilizing the oscillatory pitching response due to the presence of valve friction. These diagrams are presented to illustrate how the addition of valve preload or stick preload affects the amplitude and phasing of R with respect to F in the oscillating system. It should be noted that the gains and the amounts of friction and preload present in these two examples have been adjusted to obtain approximately neutral oscillatory stability in both cases. As shown in figure 17(a), valve preload produces a considerable lag of the resultant driving force which is destabilizing but it also causes a large amplitude reduction which is stabilizing. In this case the net result is a small stabilizing effect. In a power control of this type the valve velocity is normally approximately 180° out of phase with the stick displacement. It follows that the stick preload force should also be roughly 180° out of phase with the valve friction force and therefore should tend to cancel the effect of valve friction. It is evident from figure 17(b) that the phasing of p_s which is about 145° behind f_v is such that it introduces a small amount of lag into the resultant driving force but produces a large reduction of the amplitude. The net result is a large stabilizing effect.

Combined Frictions Plus Preload

The stabilizing contributions of stick friction, stick preload, and valve preload when added separately to a control system which had insufficient stability can also be realized when these forces are present in combination. Figures 18 to 20 present time histories and vector diagrams which illustrate this point. Consistent with results previously discussed, either preload provided an increment of stability in the presence of combined frictions. The greatest stability increment came from the stick preload.

The vector diagrams of figure 19 again represent very lightly damped systems. It is of interest to note in figure 19(a) that because valve preload lags stick friction it causes a slightly greater reduction in the amplitude of the driving force R than does an equal amount of stick friction. Control-system lag, flexibility, or lost motion amplify this effect; in this respect the relative effectiveness of valve preload is increased. The data of figure 20 represent a system which has 1/2 pound of both stick and valve friction plus 1 pound of both stick and valve preload. The examples shown are for corrections of 0.05 and 0.10 radian for which the response was at least tolerable with adequate stability. However, an error of about 0.03 radian was the threshold for control motion with the linear pseudopilot adjusted to $K_\theta = 100$ and $K_{\dot{\theta}} = 25$.

Effects of Varying System Parameters

Gearing between stick and valve.- The destabilizing influence of a given amount of valve friction is modified by varying the mechanical advantage between valve and stick. For a constant K_a the mechanical advantage is determined by the value of K_b. In the case of figure 12(a) the friction forces felt at the control stick grip were 1/2 pound from stick pivot friction and 1/2 pound from friction at the servo valve and the value of K_b was 0.4. In the cases shown in figure 21 K_b was adjusted to values of 0.1 and 0.8. For $K_b = 0.1$ with no change in valve friction measured at the valve, the valve friction as measured at the stick grip was reduced to 1/8 pound and therefore the effect of valve friction on the stability of the system was largely compensated for by the presence of 1/2 pound of stick friction. Conversely, the stability of the response was noticeably reduced and the amplitude of oscillation was doubled when K_b was increased from 0.4 to 0.8 and resulted in doubling the force at the stick grip due to valve friction.

The gearing constant K_b also affects the power control system time constant in inverse proportion. Thus, the reduction in valve friction by the expedient of lowering K_b can be obtained only at the expense of increased power control lag. The variations in K_b noted above provided time constant variations from 0.025 second to 0.20 second. Variations of this magnitude did not appear to have any important effect on system performance.

Pseudopilot lag, $\frac{1}{(1 + \tau_p p)^2}$.- The value of 0.15 second for the two equal pseudopilot time constants τ_p was selected as being fairly representative of a human pilot. However, some runs were made to show the effect of varying τ_p and the time histories of figure 22 are typical examples of this group of runs. The influence of τ_p-variation was marked only when the response was oscillatory. In figure 22, which illustrates oscillatory response due to the presence of valve friction, the amplitude of the oscillation is shown to vary directly with the lag for τ_p values from 0.075 second to 0.3 second.

Valve damping and centering.- The time histories of figures 23 and 24 illustrate the negligible effect of reducing C_v and K_v from the standard values to zero on the pitching response of the closed-loop system. This effect was negligible whether or not the response was oscillatory when the value of either C_v or K_v or both was reduced to zero. As has been previously stated, valve damping C_v is felt at the stick as the practical equivalent of control-system inertia. Likewise valve centering K_v becomes the practical equivalent of damping at the stick.

Moment of inertia of the control system (with and without damping).- The increasing use of response-feel systems which incorporate double bobweights results in increased moment-of-inertia values for longitudinal control systems. In this investigation the importance of inertia variations was found to be dependent on the amount of damping present. The effect of a tenfold increase in the moment of inertia of the simulated control system with the values of C_s and K_v set equal to zero is shown in figure 25. For the standard value of I (±0.8) and with no friction or preload in the system, the reduction of control system damping to zero did not visibly affect the airplane pitching response. (Compare fig. 25(a) with fig. 4(a).) However, as a result a small-amplitude oscillation was then superimposed on the response of the stick and valve. As the moment of inertia of the longitudinal control system was increased from 0.8 to 8.0, the pitching response of the airplane at the frequency of the control system was increased. The amplitude of the control oscillation increased until it became divergent at I = 8. In this case a gradually divergent oscillation was superimposed on the pitching response of the airplane.

Figure 26 illustrates the effect of steps taken to improve the response for the high inertia case of figure 25(d). Figure 26(a) shows that the addition of 1/2 pound of stick friction made the oscillations of θ and δ converge. However, as is shown in figure 26(b), replacing the standard stick damping C_s = 44.8 ft-lb/radian/sec (with no friction) caused the response with I = 8 to become satisfactory.

Control-Free Response

Several runs (not shown) were made to check the stick-free response of the simulated airplane to a pulse-type disturbance of the stick. These runs were made with friction and preload combinations which had resulted in hunting oscillations when the pseudopilot was attempting to control the motion. (It should be noted that, although the stick was free, the elevator position was not affected by aerodynamic loading because of the irreversible servomechanism.) The stick-free response to impulse was well damped in all cases. For some cases in which friction stopped the stick in a deflected position, the pitch-attitude response of the airplane became a steady divergence. Indications from these results are that the existence of the control difficulties studied herein cannot be established from a stick-free type of investigation.

CONCLUSIONS

An analog-computer investigation has been made of the effects of control friction on the closed-loop pitch-attitude response of a system which included an airplane with powered controls and a pseudopilot having linear response. This linear pilot response was not only advantageous in providing ease of simulation but also in providing results that represent a desirably simple mode of control for the automatic or human pilot. The condition that adequate control shall be possible with linear pilot response is believed to be a critical requirement for desirable control characteristics. The following conclusions, which verify and extend the results of previous investigations, can be made:

1. Even a few ounces of friction at the servo valve caused oscillatory response. Approximate vector diagrams of hunting oscillations produced by the presence of valve friction illustrate that the increment of pitching moment due to valve friction was nearly in phase with airplane pitching velocity. Valve friction therefore acted as a driving force for the oscillation.

2. The effects of a pound or so of friction at the stick pivot, preload at the stick pivot, or preload at the servo valve were at least tolerable as no marked deterioration of stability occurred even though increases in steady-state error did result.

3. Stick friction, stick preload, or valve preload could be used to compensate in part for the undesirable effects of valve friction. Provided that the cycling of the stick encompassed the trim position, stick preload was very effective in canceling out the effects of valve friction because it was nearly 180° out of phase with valve friction. The effectiveness of stick friction or valve preload in this regard was considerably less but could be useful for compensation during out-of-trim operation for which stick preload would be ineffective.

4. From a simple extension of the results of this investigation, it is evident that the stabilizing effects of stick friction or stick preload (in the presence of valve friction) would be reduced by lost motion or flexibility in the control linkages. The stabilizing effects of valve preload would not be adversely affected by flexibility or lost motion.

5. Valve-friction effects can be minimized by reducing the gearing between the elevator-deflection error signal and the valve. This modification increases the lag of the control system and therefore must be used in moderation.

6. The results of this investigation show that it is not possible to define adequately the characteristics of an airplane and its control system by investigating the open-loop or stick-free response. The results show that a closed-loop simulation of pilot-airplane response is required to detect the effects of system nonlinearities such as control friction.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., September 4, 1957.

REFERENCES

1. Mathews, Charles W.: Analog Study of the Effects of Various Types of Control Feel on the Dynamic Characteristics of a Pilot-Airplane Combination. NACA RM L55F01a, 1955.

2. Phillips, William H., Brown, B. Porter, and Matthews, James T., Jr.: Review and Investigation of Unsatisfactory Control Characteristics Involving Instability of Pilot-Airplane Combination and Methods for Predicting These Difficulties From Ground Tests. NACA TN 4064, 1957. (Supersedes NACA RM L53F17a.)

3. Phillips, William H.: Investigation of Effect of Reduction of Valve Friction in a Power Control System by Use of a Vibrator. NACA RM L55E18a, 1955.

4. Brown, B. Porter: Ground Simulator Studies of the Effects of Valve Friction, Stick Friction, Flexibility, and Backlash on Power Control System Quality. NACA TN 3998, 1957.

5. Anon.: Flying Qualities of Piloted Airplanes. Military Specification, MIL-F-8785 (ASG), Sept. 1, 1954; Amendment-1, Oct. 19, 1954.

TABLE I

EFFECTS OF FRICTION

Figure	Type of figure: Time history	Type of figure: Vector diagram	f_s, lb	f_v, lb	P_a, lb	P_v, lb	$\Delta\theta$, radian	K_θ, lb/radian	$K_{\dot\theta}$, lb radian/sec	K_θ, lb/radian	K_a	K_b	K_c	T_p, sec	K_v, ft-lb radian	C_v, ft-lb radian/sec	K_s, ft-lb radian	C_s, ft-lb radian/sec	I, slug-ft^2	Initial overshoot, percent	θ T_y, sec	f_{as}, percent	Hunting oscillation	Amplitude, ± percent $\Delta\theta$	Remarks
4(a)	x						0.025	100	25	80	1	0.4	50	0.15	573	100	625	44.8	0.8	0	5	5	No		
4(b)	x						.025	200	50	80	1	.4	50	.15	573	100	625	44.8	.8	40	3	<2	Marginal	Marginal	Convergent
5(a)		x					.025	200	50	80	1	.4	50	.15	573	100	625	44.8	.8	40	5	<2	Marginal	Marginal	Convergent
6(a)	x		1/2				0.025	100	25	80	1	0.4	50	0.15	573	100	625	44.8	0.8	15	>12	<20	No		
6(b)	x		1				.025	100	25	80	1	.4	50	.15	573	100	625	44.8	.8	25	5	40	No		
7(a)	x		1/2				.0125	100	25	80	1	.4	50	.15	573	100	625	44.8	.8	20	5	30	No		
7(b)	x		1/2				.10	100	25	80	1	.4	50	.15	573	100	625	44.8	.8	0	7	2	No		
8(a)	x		3				.025	100	25	80	1	.4	50	.15	573	100	625	44.8	.8	70	6	70	No		No response
8(b)	x		3				.025	150	37.5	80	1	.4	50	.15	573	100	625	44.8	.8	60	5	40(-)	No		
8(c)	x		3				.025	200	50	80	1	.4	50	.15	573	100	625	44.8	.8	50	4	0	Yes	40	
5(b)		x	1				.025	150	37.5	80	1	.4	50	.15	573	100	625	44.8	.8						
9(a)	x			1/2			0.025	100	25	80	1	0.4	50	0.15	573	100	625	44.8	0.8	15	5	<5	Yes	25	
9(b)	x			1			.025	100	25	80	1	.4	50	.15	573	100	625	44.8	.8	30	5	0	Yes	50	
10(a)		x		1/2			.025	100	25	80	1	.4	50	.15	573	100	625	44.8	.8	30	5	0	Yes	50	
11(a)	x			1			.0125	100	25	80	1	.4	50	.15	573	100	625	44.8	.8	0	3	50	Yes	50	
11(b)	x			1/2			.10	100	25	80	1	.4	50	.15	573	100	625	44.8	.8	4	5	20	Yes	5	
12(a)	x		1/2	1/2			0.025	100	25	80	1	0.4	50	0.15	573	100	625	44.8	0.8	30	5	<10	Yes	35	
12(b)	x		1	1/2			.025	100	25	80	1	.4	50	.15	573	100	625	44.8	.8	35	>20	<20	No		
10(b)		x	1/2	1/2			.025	100	25	80	1	.4	50	.15	573	100	625	44.8	.8	30	5	<10	Yes	35	

TABLE II

PARTIAL COMPENSATION FOR FRICTION EFFECTS WITH PRELOADS

Figure	Type of figure		f_s, lb	f_v, lb	P_s, lb	P_v, lb	$\Delta\theta$, radian	K_θ, lb/radian	$K_{\dot{\theta}}$, $\frac{\text{lb}}{\text{radian/sec}}$	$K_{\ddot{\theta}}$, lb/radian	K_a	K_b	K_c	τ_p, sec	K_v, $\frac{\text{ft-lb}}{\text{radian}}$	C_v, $\frac{\text{ft-lb}}{\text{radian/sec}}$	K_s, $\frac{\text{ft-lb}}{\text{radian}}$	C_s, $\frac{\text{ft-lb}}{\text{radian/sec}}$	I, slug-ft^2	Initial θ overshoot, percent	T_5, sec	ϵ_{ss}, percent	Hunting oscillation	Amplitude, ± percent $\Delta\theta$	Remarks
	Time history	Vector diagram																							
13(a)	x				1/2		0.025	100	25	80	1	0.4	50	0.15	573	100	625	44.8	0.8	0	>16	<20	No		
13(b)	x				1		.025	100	25	80	1	.4	50	.15	573	100	625	44.8	.8	0	>18	<40	No		
14(a)	x					1/2	0.025	100	25	80	1	0.4	50	0.15	573	100	625	44.8	0.8	12	5	2	No		
14(b)	x					1	.025	100	25	80	1	.4	50	.15	573	100	625	44.8	.8	20	3.5	4	No		
15(a)	x		1/2		1		0.025	100	25	80	1	0.4	50	0.15	573	100	625	44.8	0.8	0	>15	<20	No		
15(b)	x			1/2		1	.025	100	25	80	1	.4	50	.15	573	100	625	44.8	.8	35	>20	<20	No		
16(a)	x			1/2	1/2		.025	100	25	80	1	.4	50	.15	573	100	625	44.8	.8	0	>20	<20	No		
16(b)	x			1/2	1		.025	100	25	80	1	.4	50	.15	573	100	625	44.8	.8	0	>20	<20	No		
17(a)		x		1/2		1	0.025	150	37.5	80	1	0.4	50	0.15	573	100	625	44.8	0.8	50	4	10	Yes		
17(b)		x		1	1/2		.025	100	25	80	1	.8	50	.15	573	100	625	44.8	.8	20	4	5	Yes		
18(a)	x		1/2	1/2		1	0.025	100	25	80	1	0.4	50	0.15	573	100	625	44.8	0.8	45	>20	<50	No		
18(b)	x		1/2	1/2	1		.025	100	25	80	1	.4	50	.15	573	100	625	44.8	.8	0	>20	<20	No		
19(a)		x	1/2	1/2		1/2	0.025	200	50	80	1	0.4	50	0.15	573	100	625	44.8	0.8	80	2	0	Yes	>250	Divergent
19(b)		x	1	1	1		.025	200	50	80	1	.4	50	.15	573	100	625	44.8	.8	50	3	10	Marginal	Marginal	Convergent
20(a)	x		1/2	1/2	1	1	0.05	100	25	80	1	0.4	50	0.15	573	100	625	44.8	.8	5	4	≤20(-)	No		
20(b)	x		1/2	1/2	1	1	.10	100	25	80	1	.4	50	.15	573	100	625	44.8	.8	4	7	2	No		

CONFIDENTIAL

TABLE III

EFFECTS OF VARYING SYSTEM PARAMETERS

Figure	Type of figure		f_s, lb	f_v, lb	p_s, lb	p_v, lb	$\Delta\theta$, radian	K_θ, lb/radian	$K_{\dot{\theta}}$, $\frac{\text{lb}}{\text{radian/sec}}$	K_s, lb/radian	K_a	K_b	K_c	τ_p, sec	K_v, $\frac{\text{ft-lb}}{\text{radian}}$	C_v, $\frac{\text{ft-lb}}{\text{radian/sec}}$	K_s, $\frac{\text{ft-lb}}{\text{radian}}$	C_s, $\frac{\text{ft-lb}}{\text{radian/sec}}$	I, slug-ft^2	Initial θ overshoot, percent	T_s, sec	ϵ_{ss}, percent	Hunting oscillation	Amplitude, ± percent $\Delta\theta$	Remarks
	Time history	Vector diagram																							
21(a)	x		1/2	1/8			0.025	100	25	80	1	0.1	50	0.15	573	100	625	44.8	0.8	25	>16	<20	No		
21(b)	x		1/2	1			.025	100	25	80	1	.8	50	.15	573	100	625	44.8	.8	40	4	<2	Yes	65	
22(a)	x			1/2			0.025	100	25	80	1	0.4	50	0.15	573	100	625	44.8	0.8	10	5 to 6	<4(-)	Yes	25	
22(b)	x			1/2			.025	100	25	80	1	.4	50	.30	573	100	625	44.8	.8	30	5 to 6	<4(-)	Yes	45	
22(c)	x			1/2			.025	100	25	80	1	.4	50	.075	573	100	625	44.8	.8	0	10	<4(-)	Yes	15	
23(a)	x			1/2			0.025	100	25	80	1	0.4	50	0.15	0	100	625	44.8	0.8	10	5 to 6	<4(-)	Yes	25	
23(b)	x			1/2			.025	100	25	80	1	.4	50	.15	0	0	625	44.8	.8	10	5 to 6	<4(-)	Yes	25	
23(c)	x			1/2			.025	100	25	80	1	.4	50	.15	573	0	625	44.8	.8	10	5 to 6	<4(-)	Yes	25	
23(d)	x			1/2			.025	100	25	80	1	.4	50	.15	573	100	625	44.8	.8	10	5 to 6	<4(-)	Yes	25	
24(a)	x		1/2	1/2	1		0.025	100	25	80	1	0.4	50	0.15	573	100	625	44.8	0.8	0	>8	<20(-)	No		
24(b)	x		1/2	1/2	1		.025	100	25	80	1	.4	50	.15	573	0	625	44.8	.8	0	>8	<20(-)	No		
24(c)	x		1/2	1/2	1		.025	100	25	80	1	.4	50	.15	0	0	625	44.8	.8	0	>8	<20(-)	No		
24(d)	x		1/2	1/2	1		.025	100	25	80	1	.4	50	.15	0	100	625	44.8	.8	0	>8	<20(-)	No		
25(a)	x						0.025	100	25	80	1	0.4	50	0.15	0	100	625	0	0.8	0	5	0	No		
25(b)	x						.025	100	25	80	1	.4	50	.15	0	100	625	0	1.6	0	5	2(-)	No		
25(c)	x						.025	100	25	80	1	.4	50	.15	0	100	625	0	4.0	0	5	0	No		
25(d)	x						.025	100	25	80	1	.4	50	.15	0	100	625	0	8.0	2	5	0	Yes	±20	Divergent
26(a)	x		1/2				0.025	100	25	80	1	0.4	50	0.15	0	100	625	0	8.0	0	5	2	No		
26(b)	x						.025	100	25	80	1	.4	50	.15	0	100	625	44.8	8.0	0	5	2	No		

CONFIDENTIAL

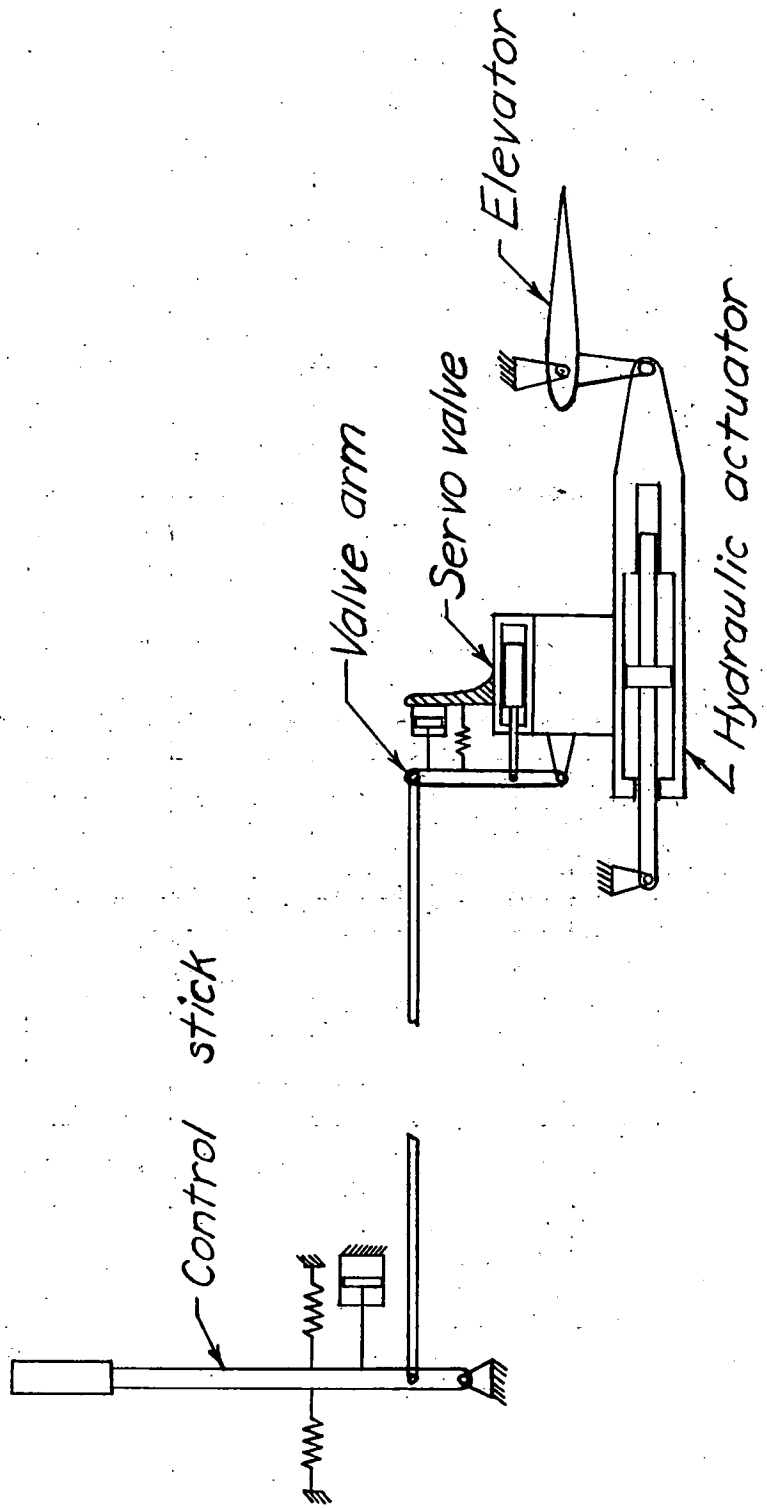


Figure 1.- Schematic diagram of simulated longitudinal control system.

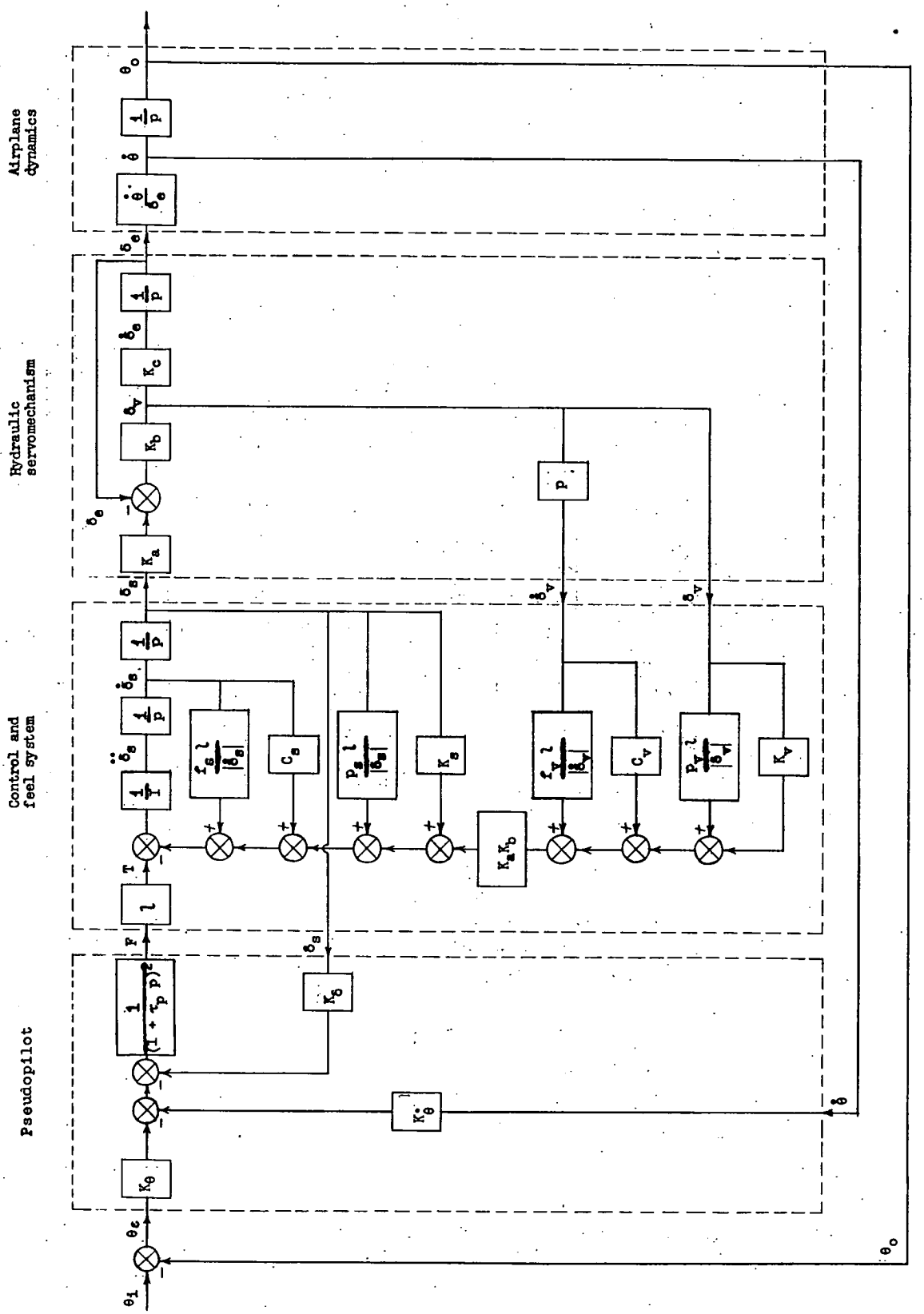
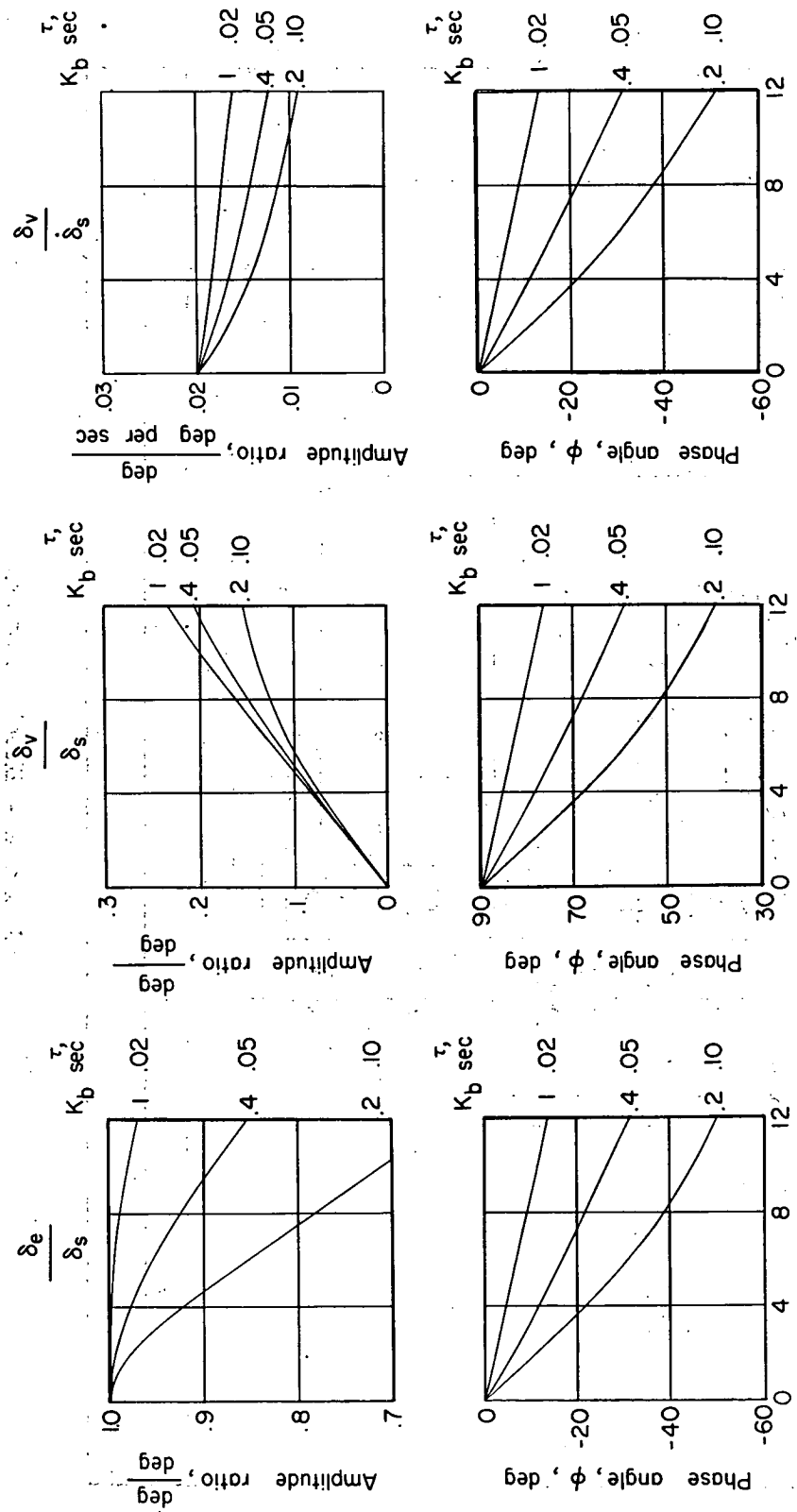


Figure 2.- Block diagram.

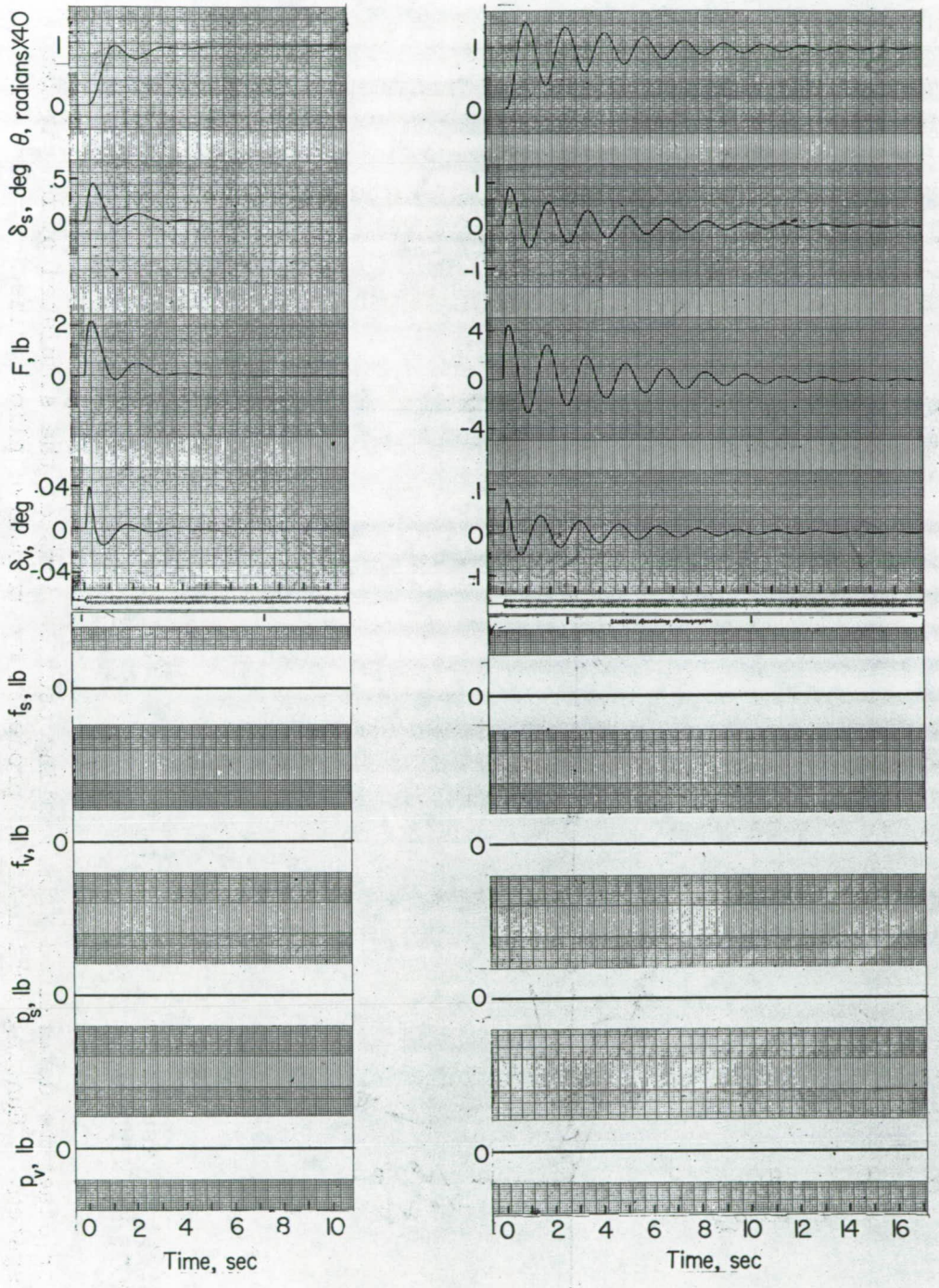


(a) $\frac{\delta_e}{\delta_s} = \frac{K_a}{1 + \frac{p}{K_b K_c}}$.

(b) $\frac{\delta_v}{\delta_s} = \frac{K_a K_b}{1 + \frac{K_b K_c}{p}}$.

(c) $\frac{\delta_v}{\delta_s} = \frac{K_a K_b}{p + K_b K_c}$.

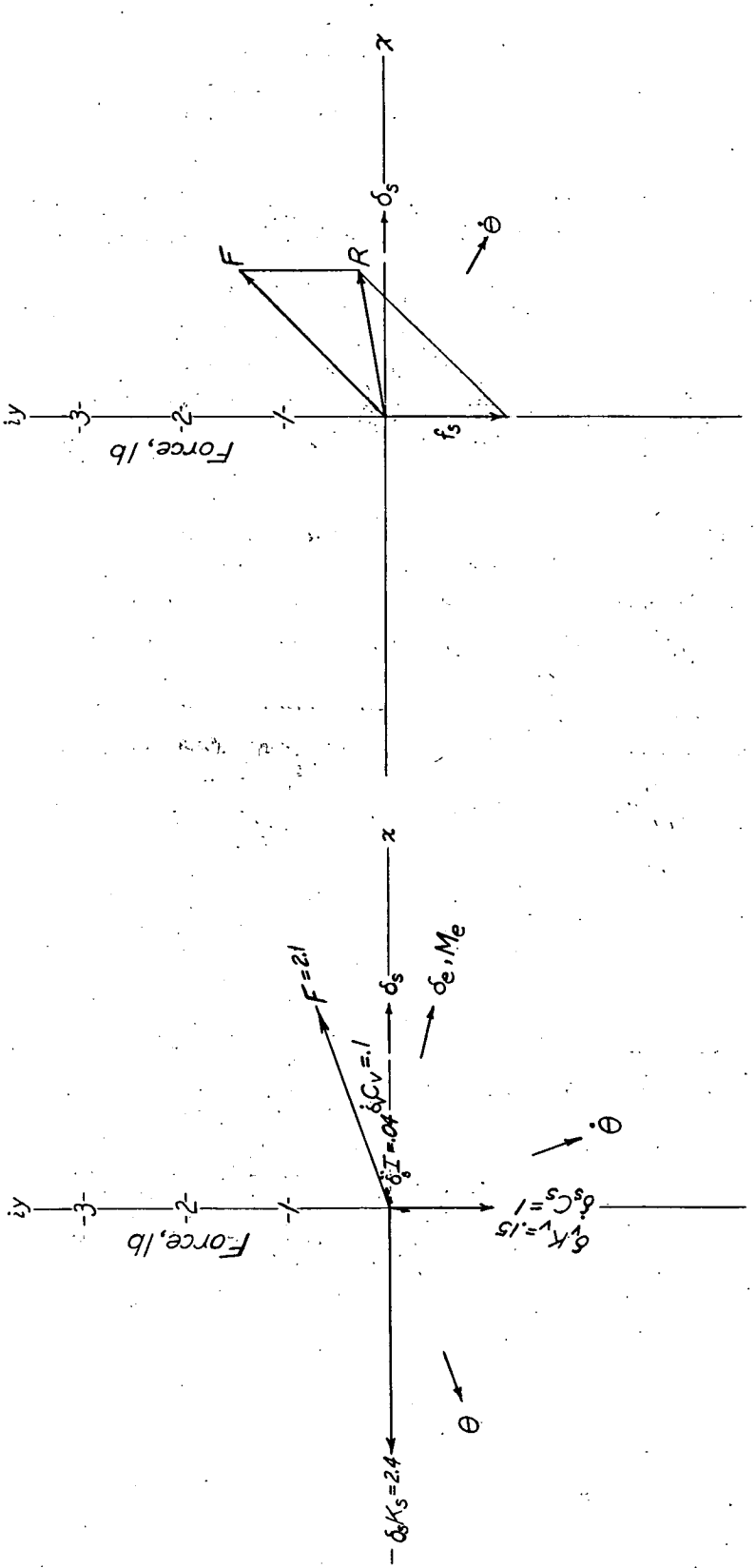
Figure 3.- Calculated frequency responses of the simulated longitudinal power control system for three pertinent transfer functions. $K_a = 1$ and $K_c = 50$.



(a) For standard gains, $K_\theta = 100$; $K_{\dot{\theta}} = 25$; $K_b = 0.4$.

(b) With increased gains, $K_\theta = 200$; $K_{\dot{\theta}} = 50$; $K_b = 0.4$.

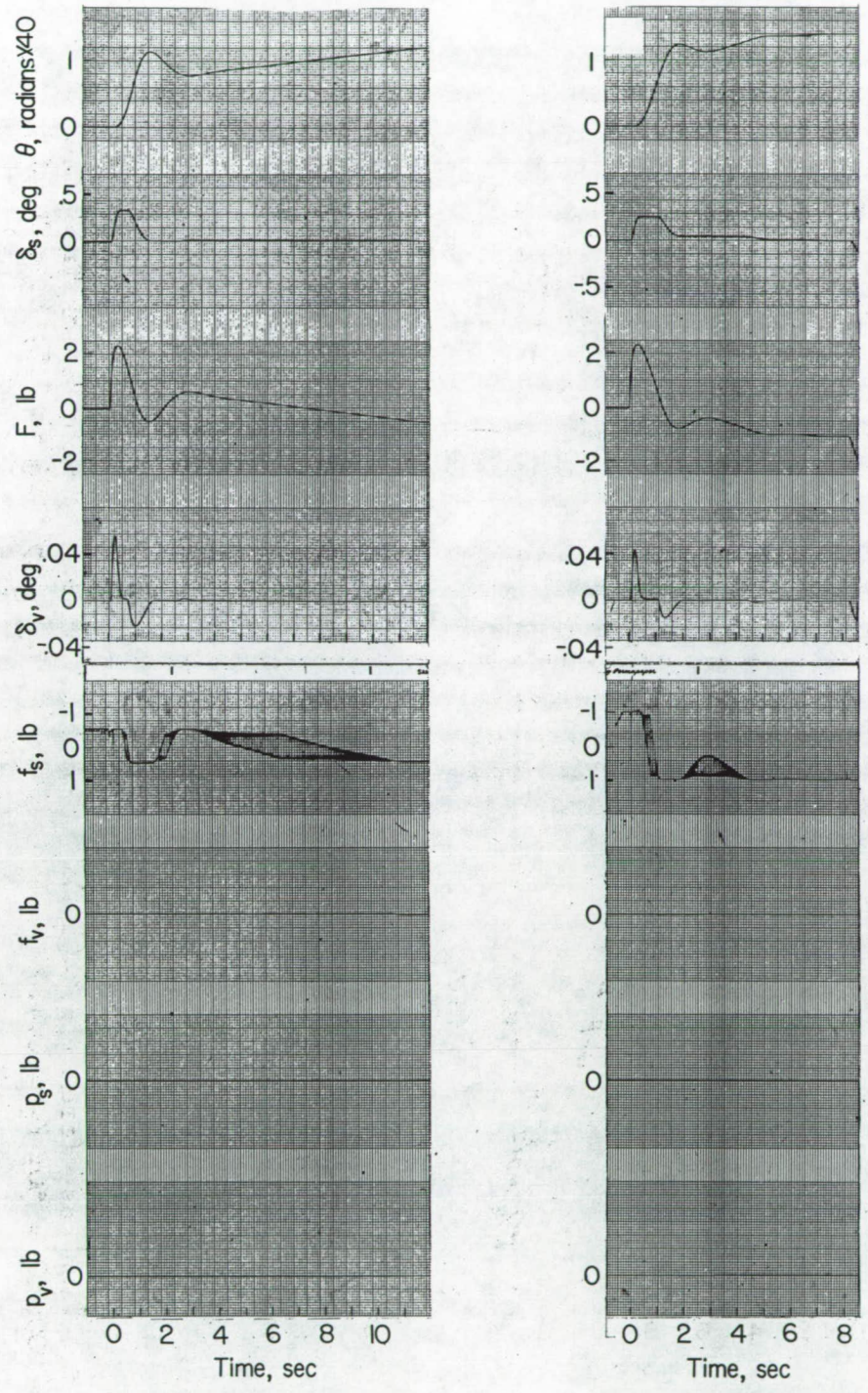
Figure 4.- Closed-loop response of the simulated pilot-airplane combination with no friction or preload.



(a) No friction case (from fig. 4(b)).
$K_\theta = 200$; $K_{\dot{\theta}} = 50$; $K_b = 0.4$.

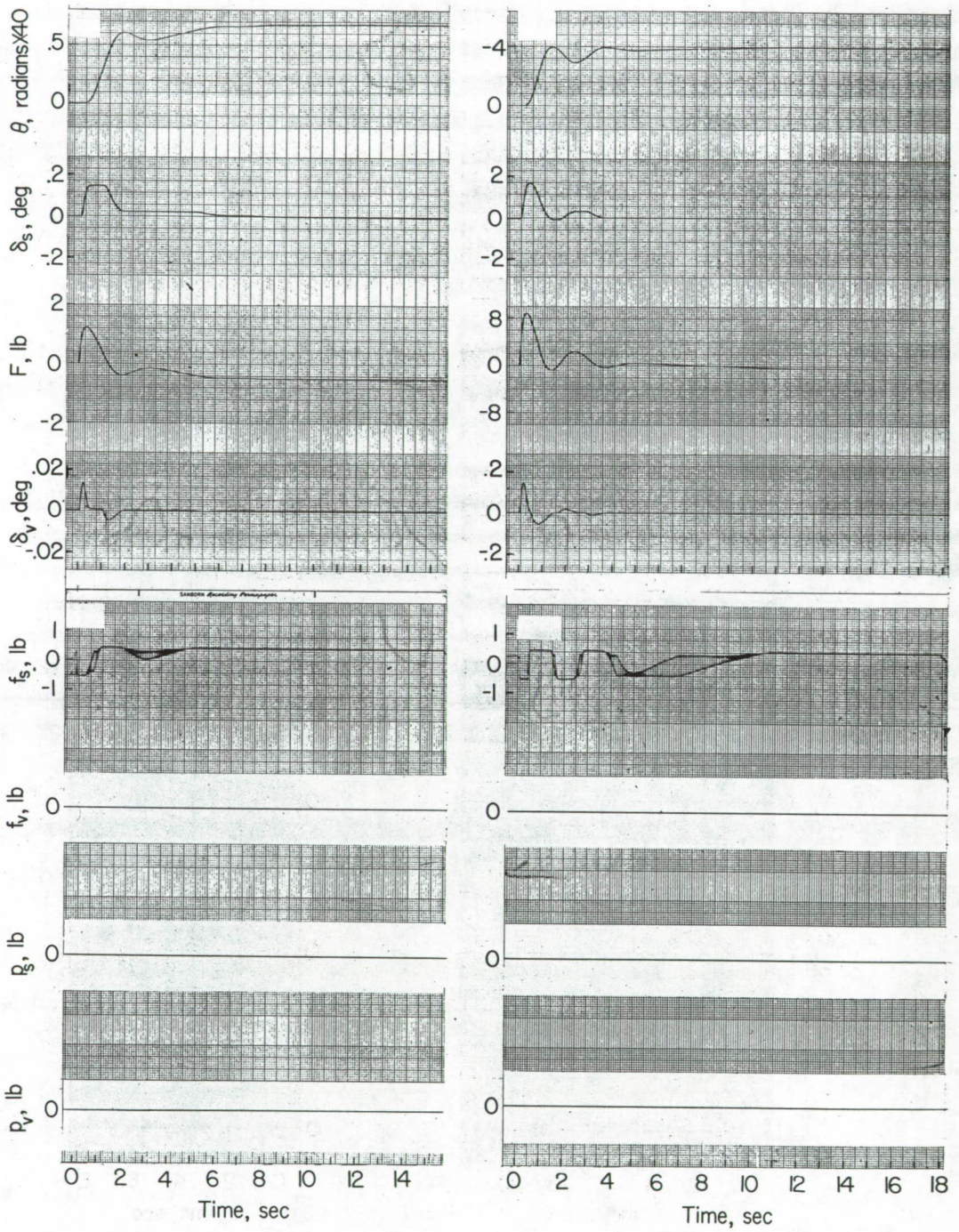
(b) With 1 pound of stick friction.
$K_\theta = 150$; $K_{\dot{\theta}} = 37.5$; $K_b = 0.4$.

Figure 5.- Vector diagrams determined from hunting oscillations of the simulated pilot-airplane system. (Hunting oscillations produced by excessive pilot gains.)



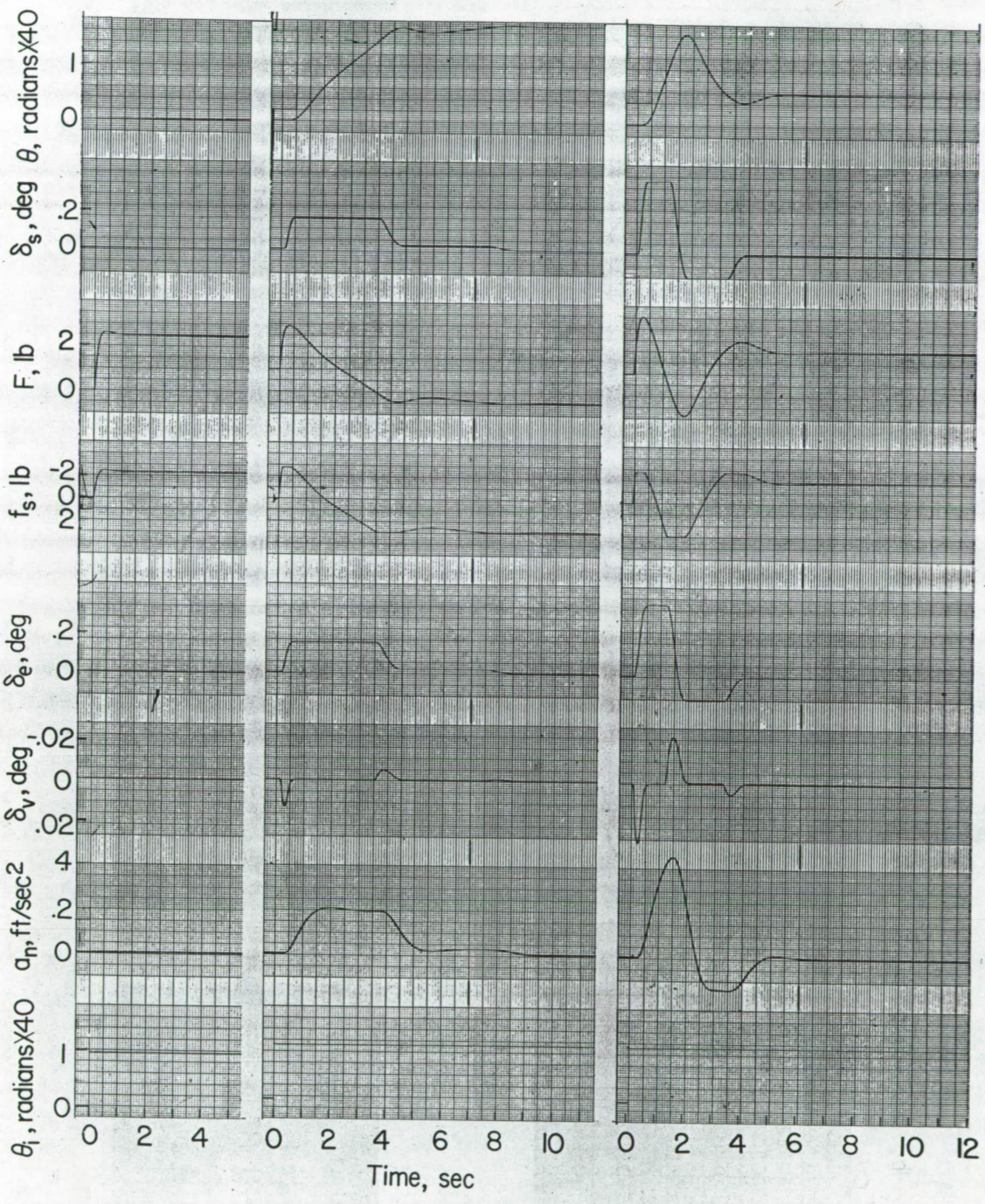
(a) f_s = 1/2 pound. (b) f_s = 1 pound.

Figure 6.- Effect of stick pivot friction on the closed-loop response.



(a) $\Delta\theta$ = 0.0125 radian. (b) $\Delta\theta$ = 0.10 radian.

Figure 7.- An illustration of the variation of the effect of stick pivot friction with magnitude of pitch attitude correction. (f_s = 1/2 pound.)

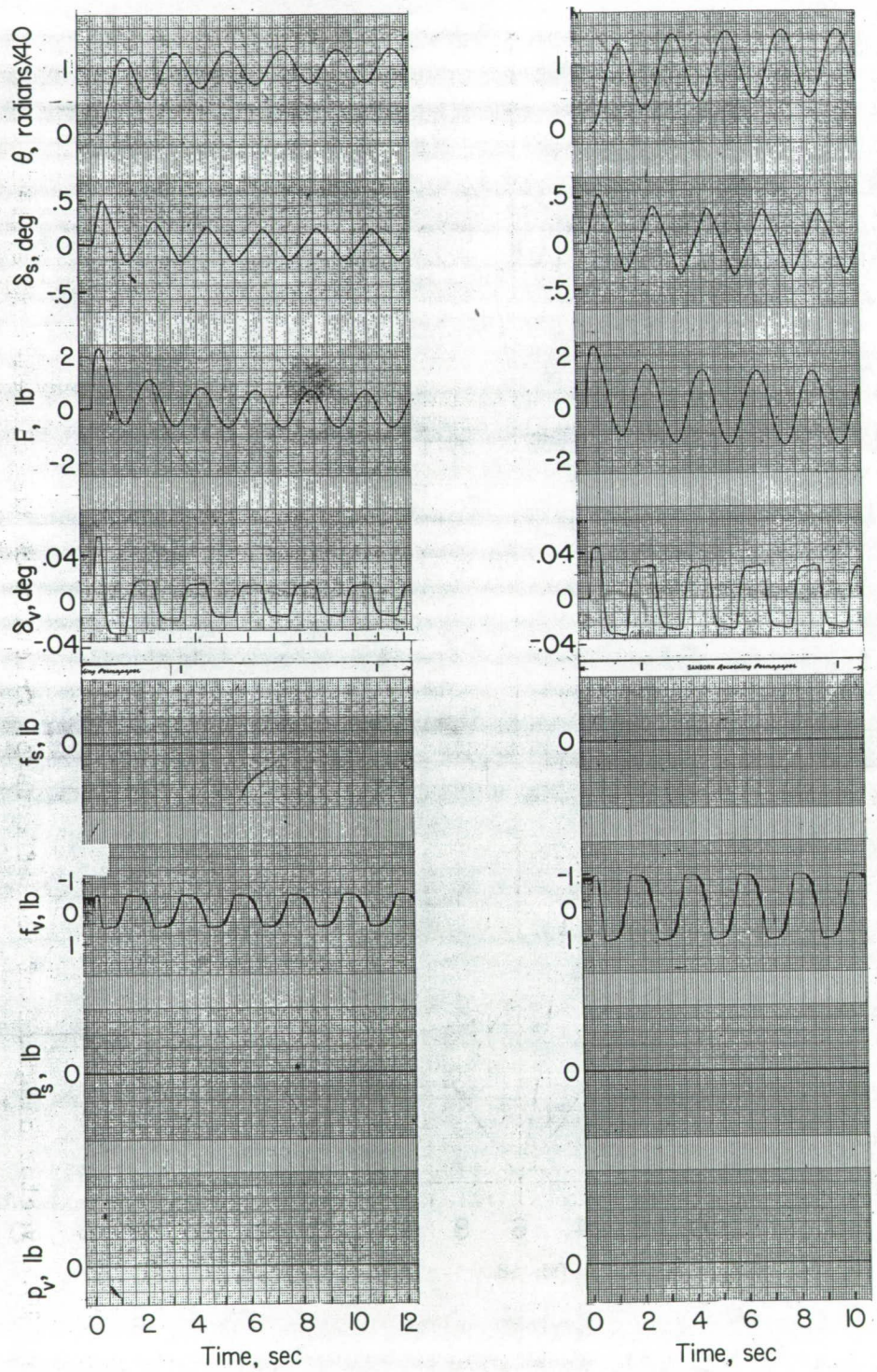


(a) $K_\theta = 100$; $K_{\dot{\theta}} = 25$; $K_b = 0.4$.

(b) $K_\theta = 150$; $K_{\dot{\theta}} = 37.5$; $K_b = 0.4$.

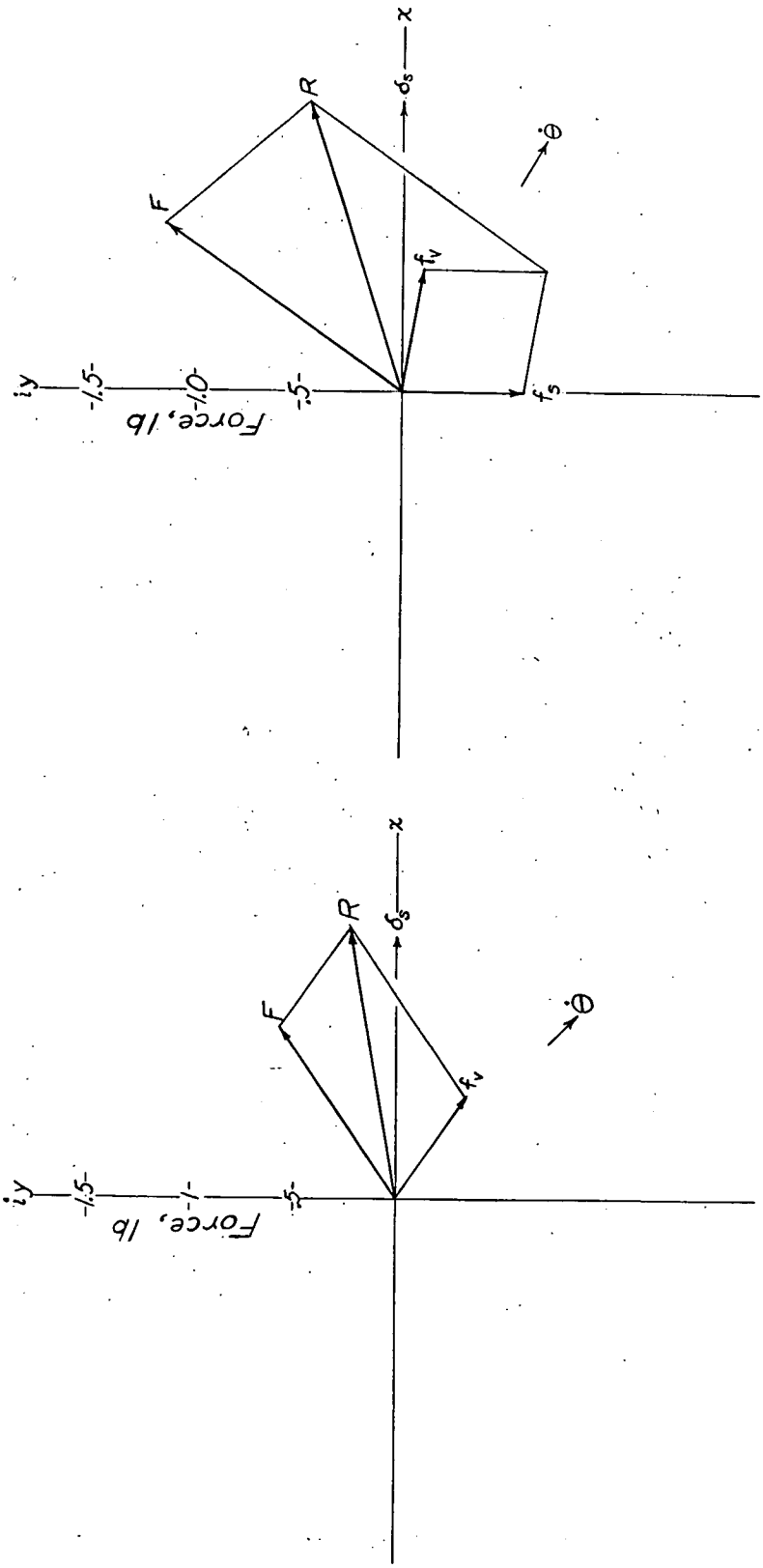
(c) $K_\theta = 200$; $K_{\dot{\theta}} = 50$; $K_b = 0.4$.

Figure 8.- An illustration of the ability of the linear pilot to overcome the effects of excess stick pivot friction (3 lb) by means of increased gains.



(a) f_V = 1/2 pound. (b) f_V = 1 pound.

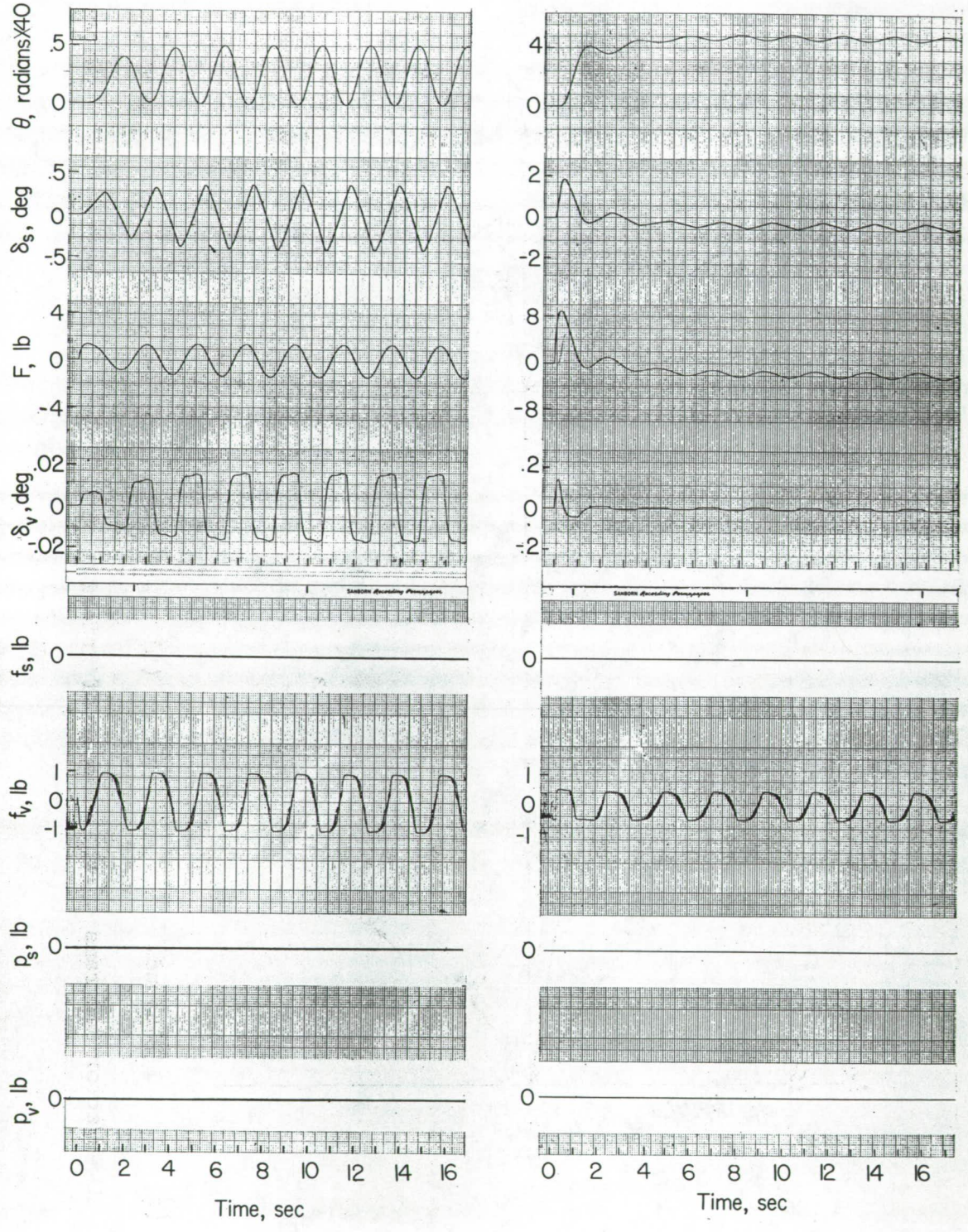
Figure 9.- The effect of servovalve friction on closed-loop response.



(a) f_v = 1/2 pound.

(b) f_s = 1/2 pound; f_v = 1/2 pound.

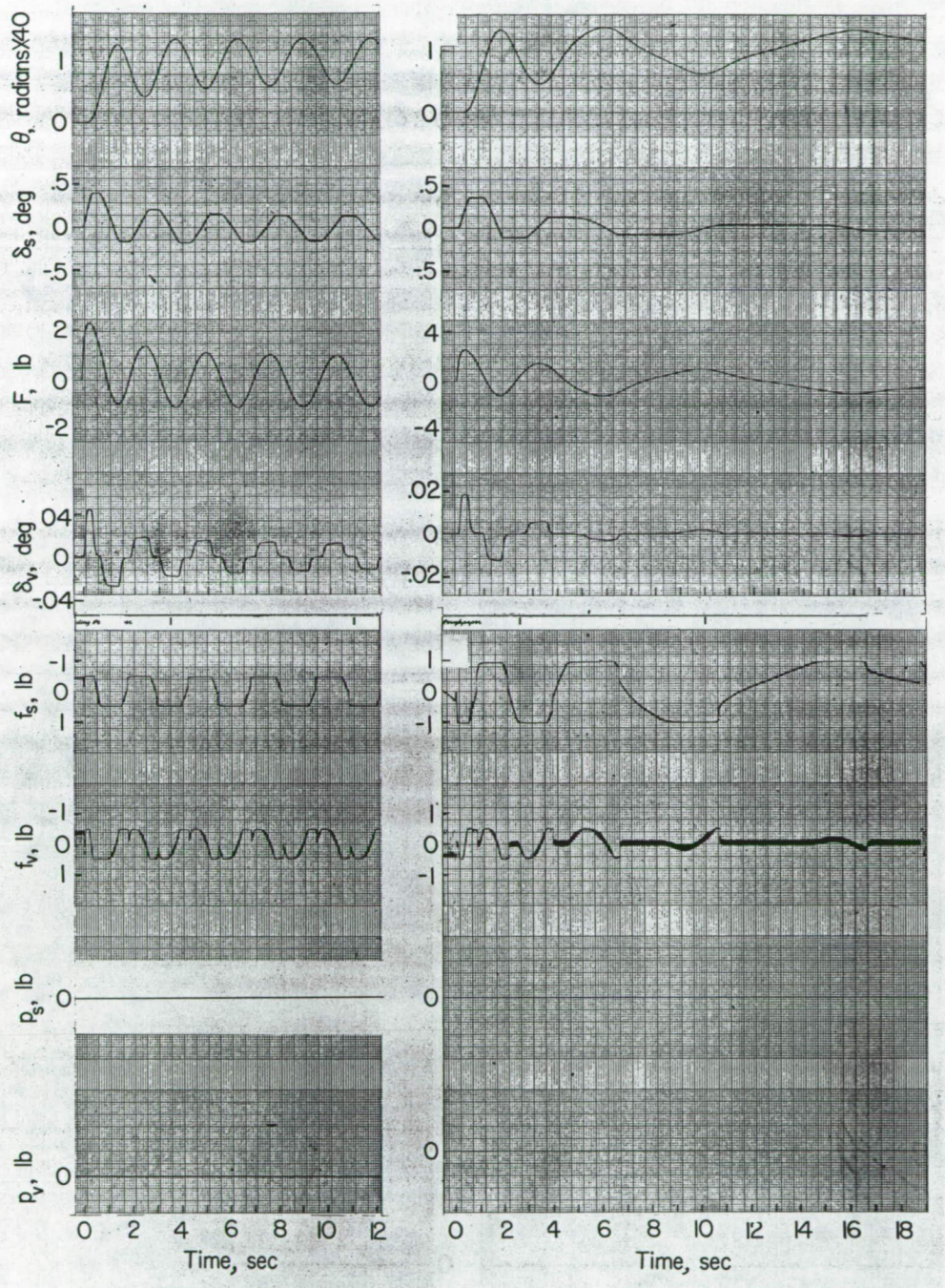
Figure 10.- Vector diagrams of the effects of friction on the closed-loop system.



(a) f_V = 1 pound; $\Delta\theta$ = 0.0125 radian.

(b) f_V = 1/2 pound; $\Delta\theta$ = 0.10 radian.

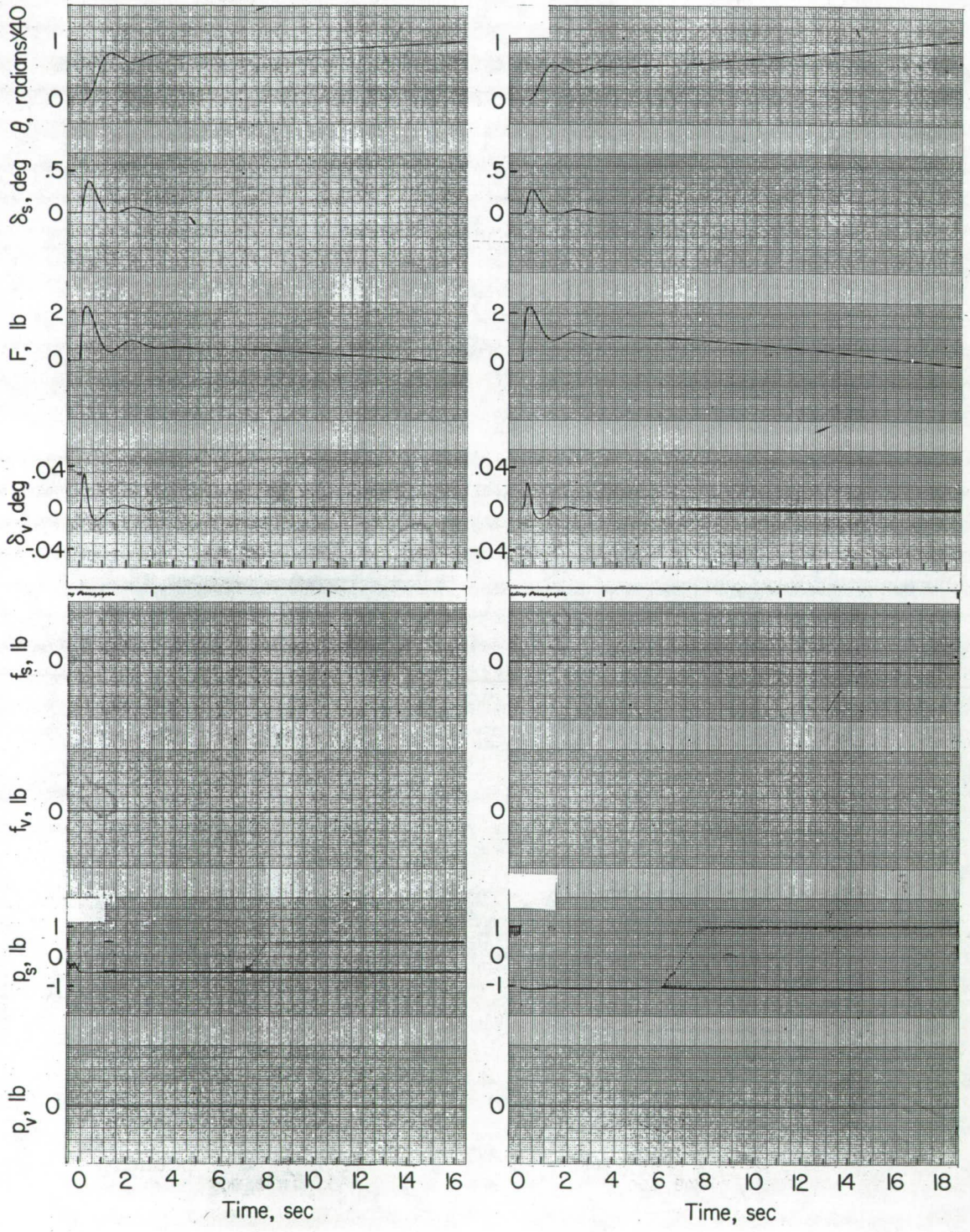
Figure 11.- An illustration of the variation of the effects of servovalve friction with the magnitude of the pitch-attitude correction.



(a) f_S = 1/2 pound; f_V = 1/2 pound.

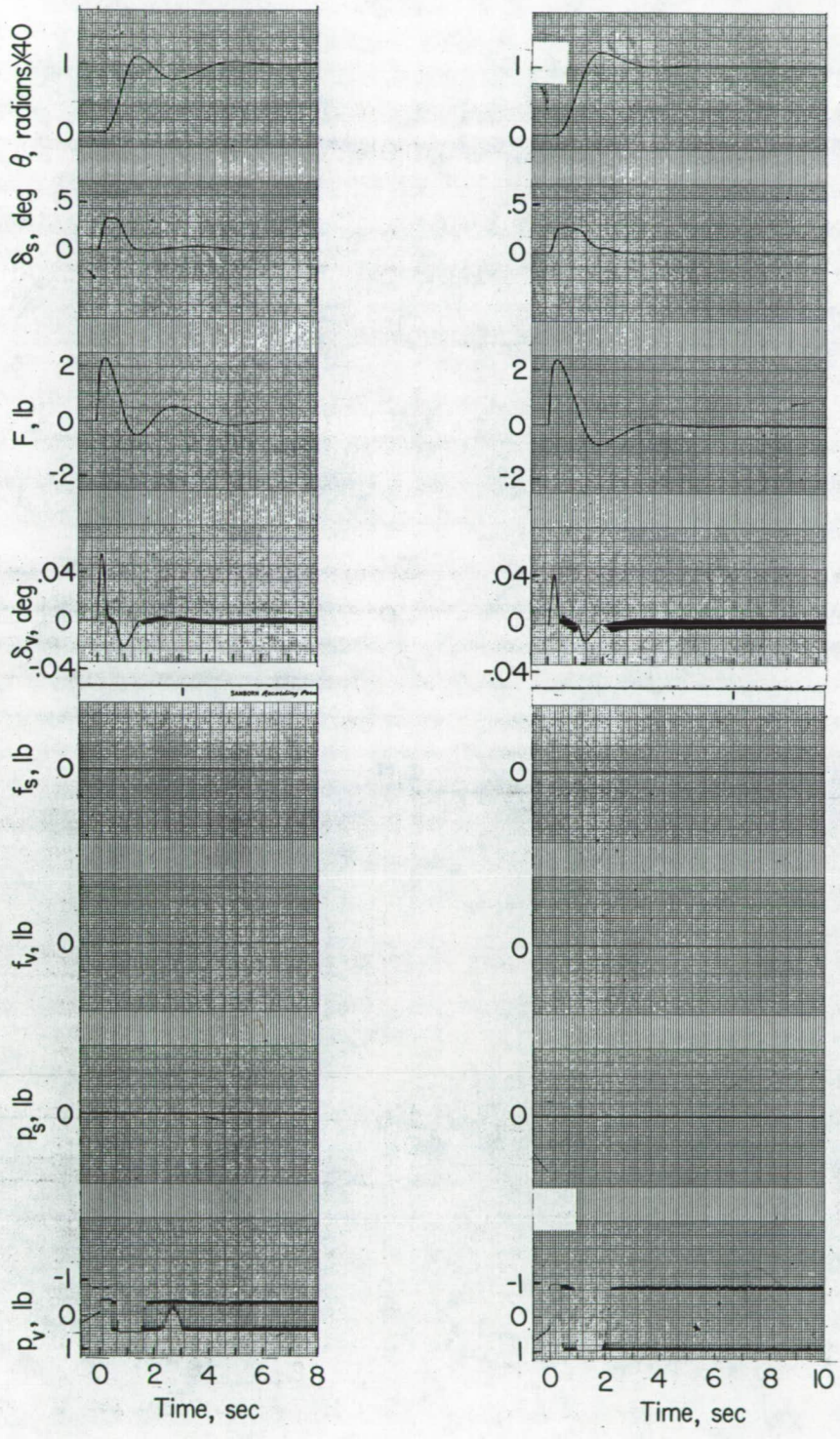
(b) f_S = 1 pound; f_V = 1/2 pound.

Figure 12.- Illustrations of the use of stick friction to compensate partially for the effects of valve friction.



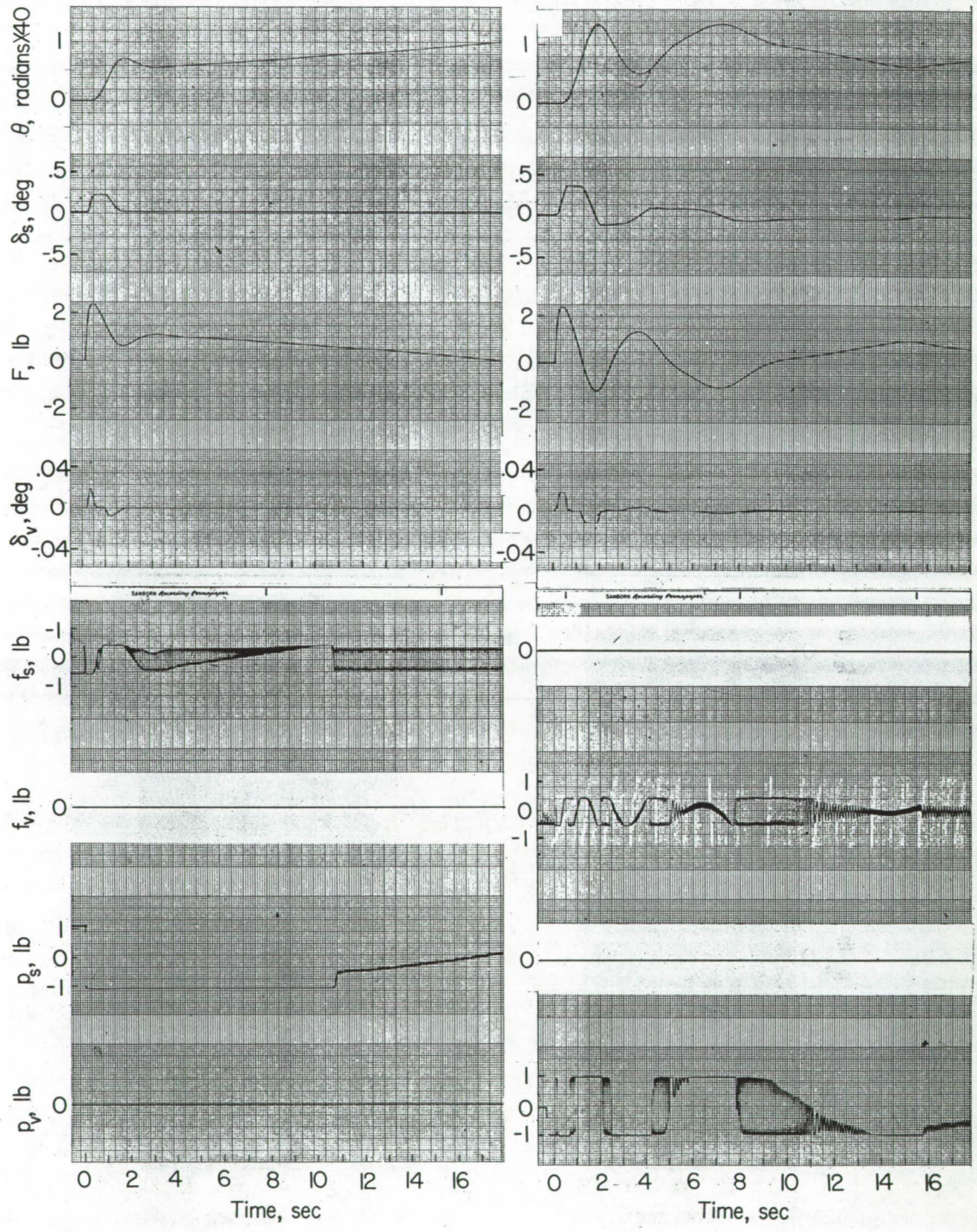
(a) p_s = 1/2 pound. (b) p_s = 1 pound.

Figure 13.- Effect of stick preload on closed-loop response.



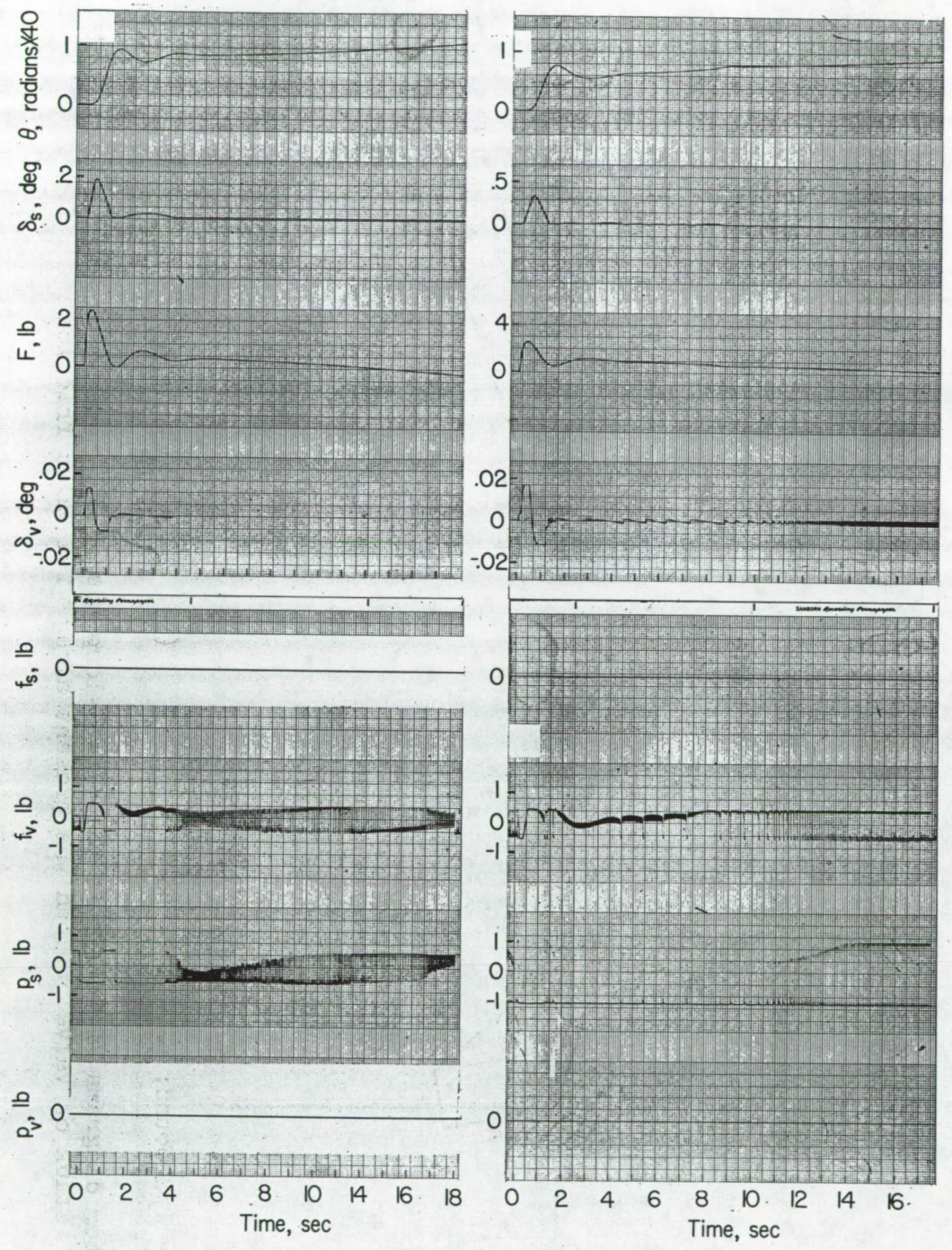
(a) p_v = 1/2 pound. (b) p_v = 1 pound.

Figure 14.- Effect of valve preload on closed-loop response.



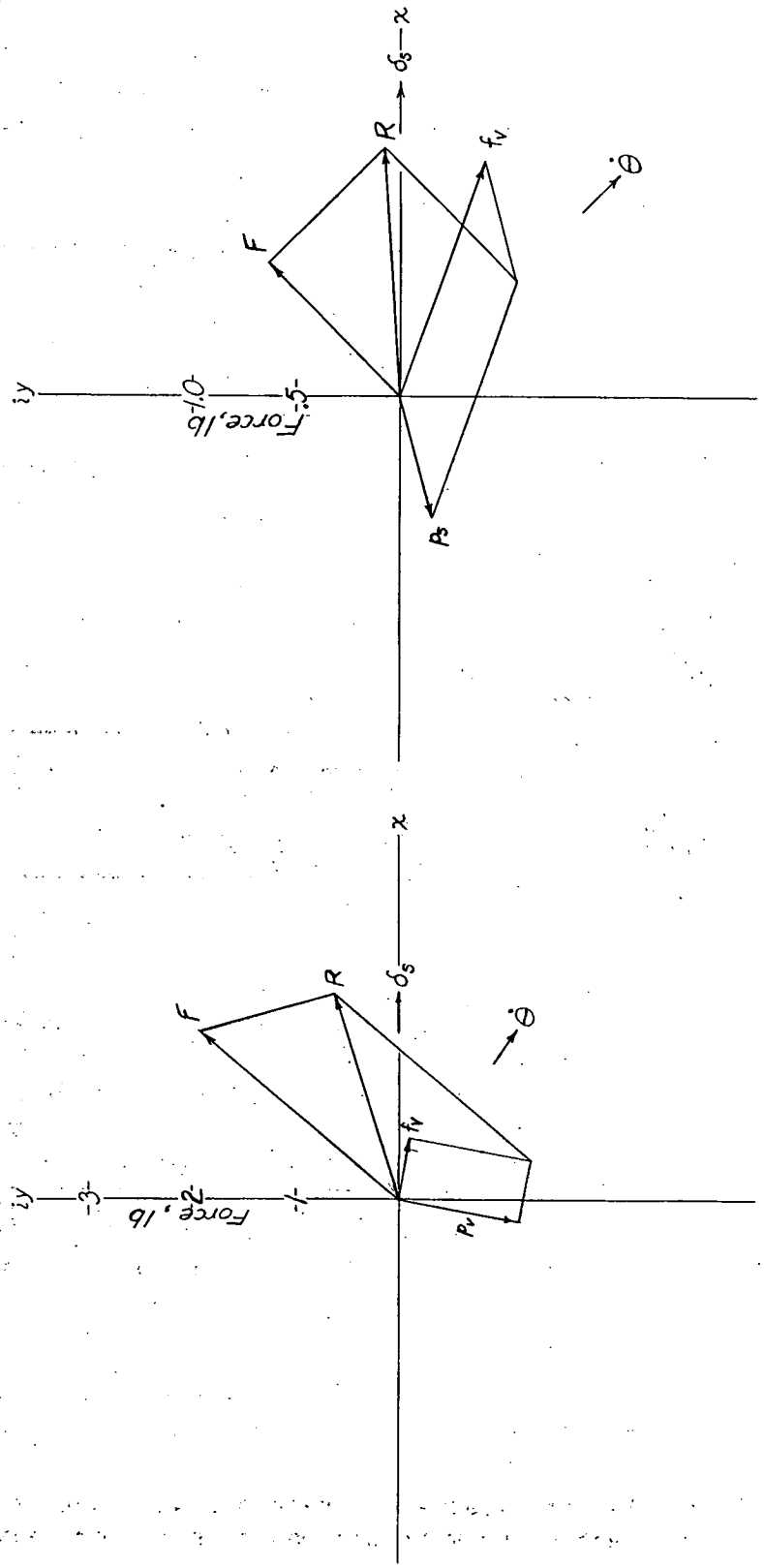
(a) f_S = 1/2 pound; p_S = 1 pound. (b) f_V = 1/2 pound; p_V = 1 pound.

Figure 15.- Effects of preloaded centering in combination with friction at the stick pivot or at the servovalve on the closed-loop response.



(a) f_v = 1/2 pound; p_s = 1/2 pound. (b) f_v = 1/2 pound; p_s = 1 pound.

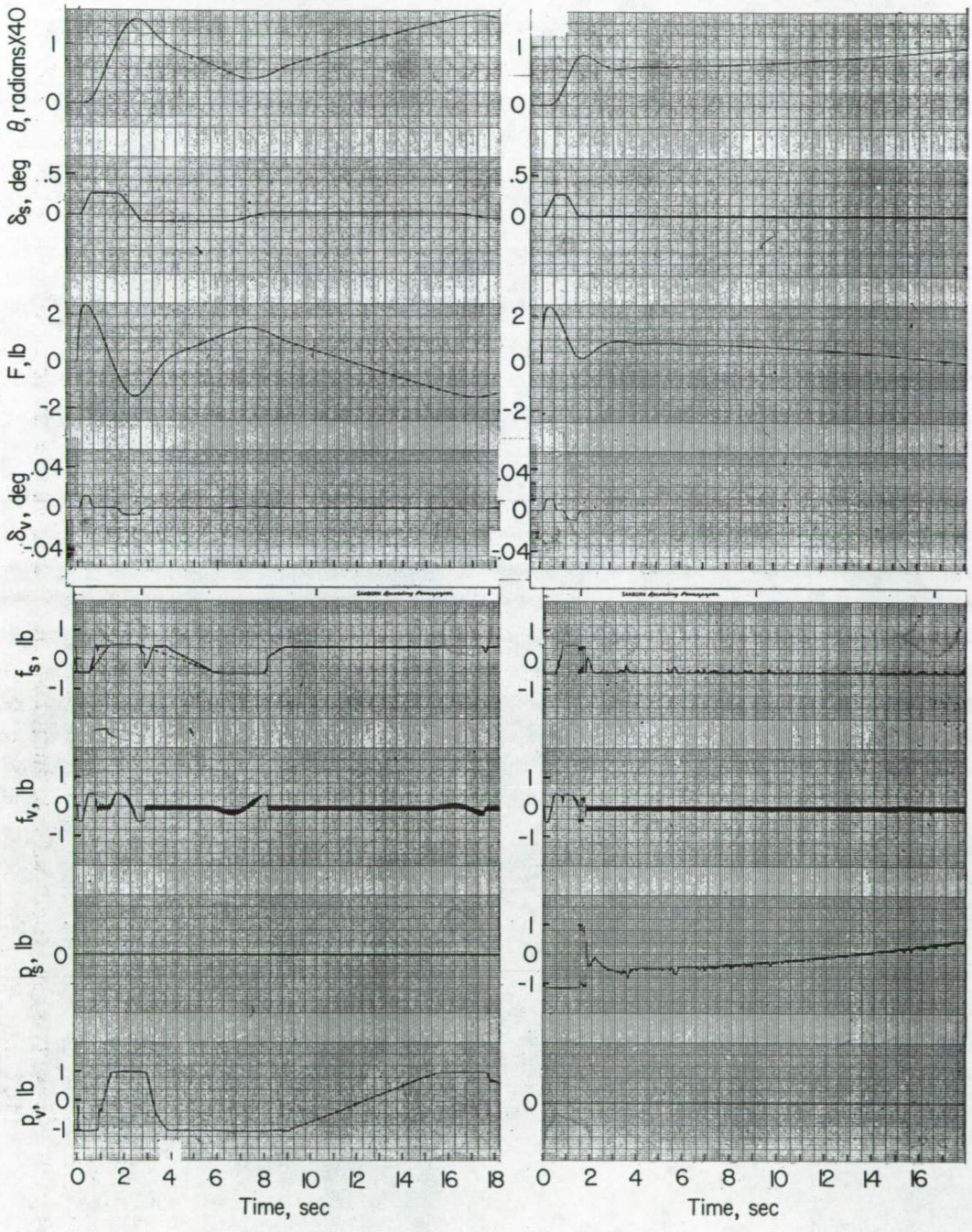
Figure 16.- Effect of preloading the stick centering springs to stabilize the closed-loop pilot-airplane system with servovalve friction present.



(a) f_v = 1/2 pound; P_v = 1 pound.

(b) f_v = 1 pound; P_s = 1/2 pound.

Figure 17.- Vector diagrams of forces in the closed-loop system during hunting oscillations when stick or valve preload was present along with valve friction.



(a) f_S = 1/2 pound; f_V = 1/2 pound; p_V = 1 pound.

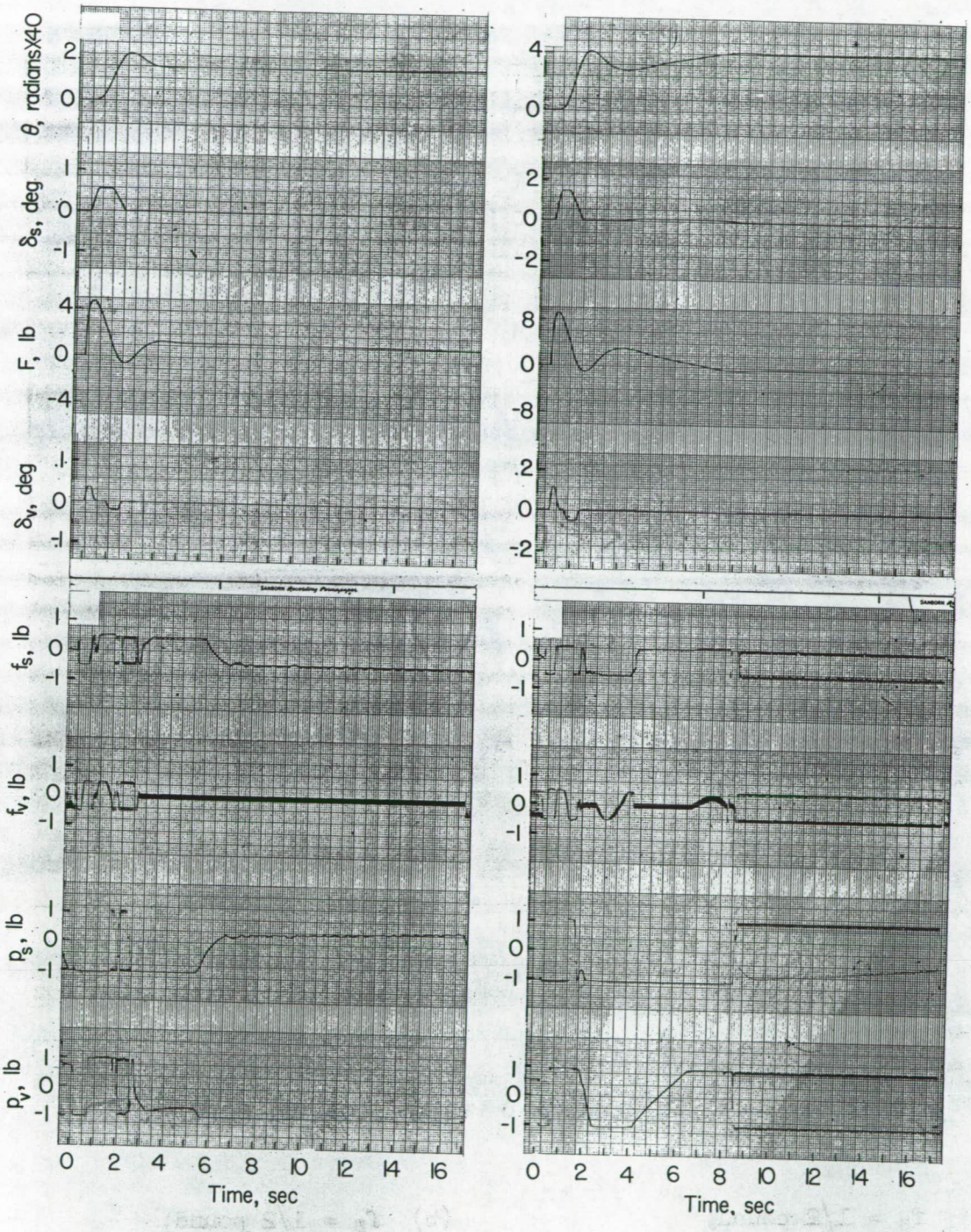
(b) f_S = 1/2 pound; f_V = 1/2 pound; p_S = 1 pound.

Figure 18.- Time histories of closed-loop response with both stick and valve friction present along with preload at stick or valve.

(a) f_s = 1/2 pound; f_v = 1/2 pound; p_v = 1/2 pound.

(b) f_s = 1 pound; f_v = 1 pound; p_s = 1 pound.

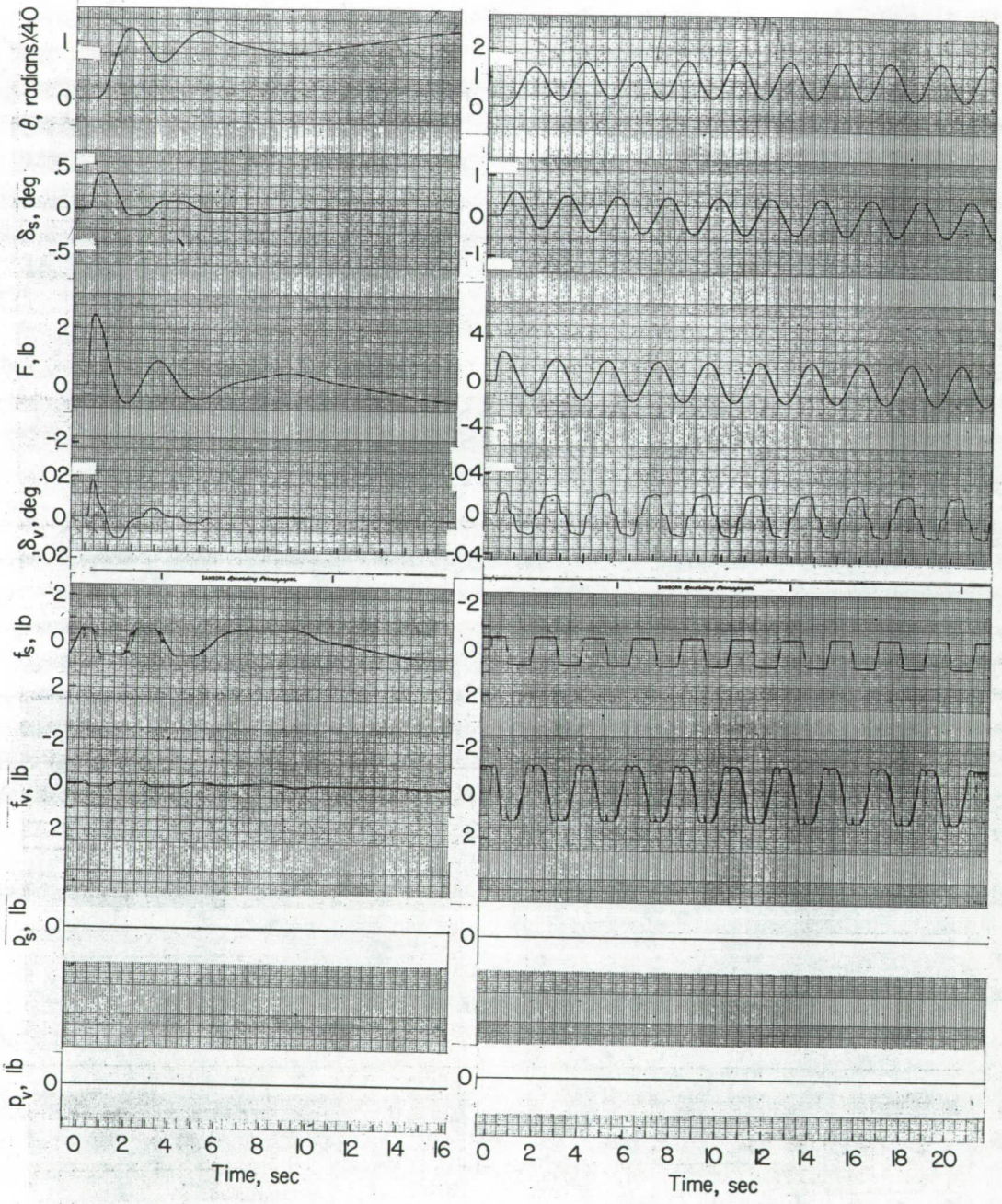
Figure 19.- Vector diagrams obtained from hunting oscillations for which both stick and valve friction were present along with stick or valve preload.



(a) $\Delta\theta$ = 0.05 radian.

(b) $\Delta\theta$ = 0.10 radian.

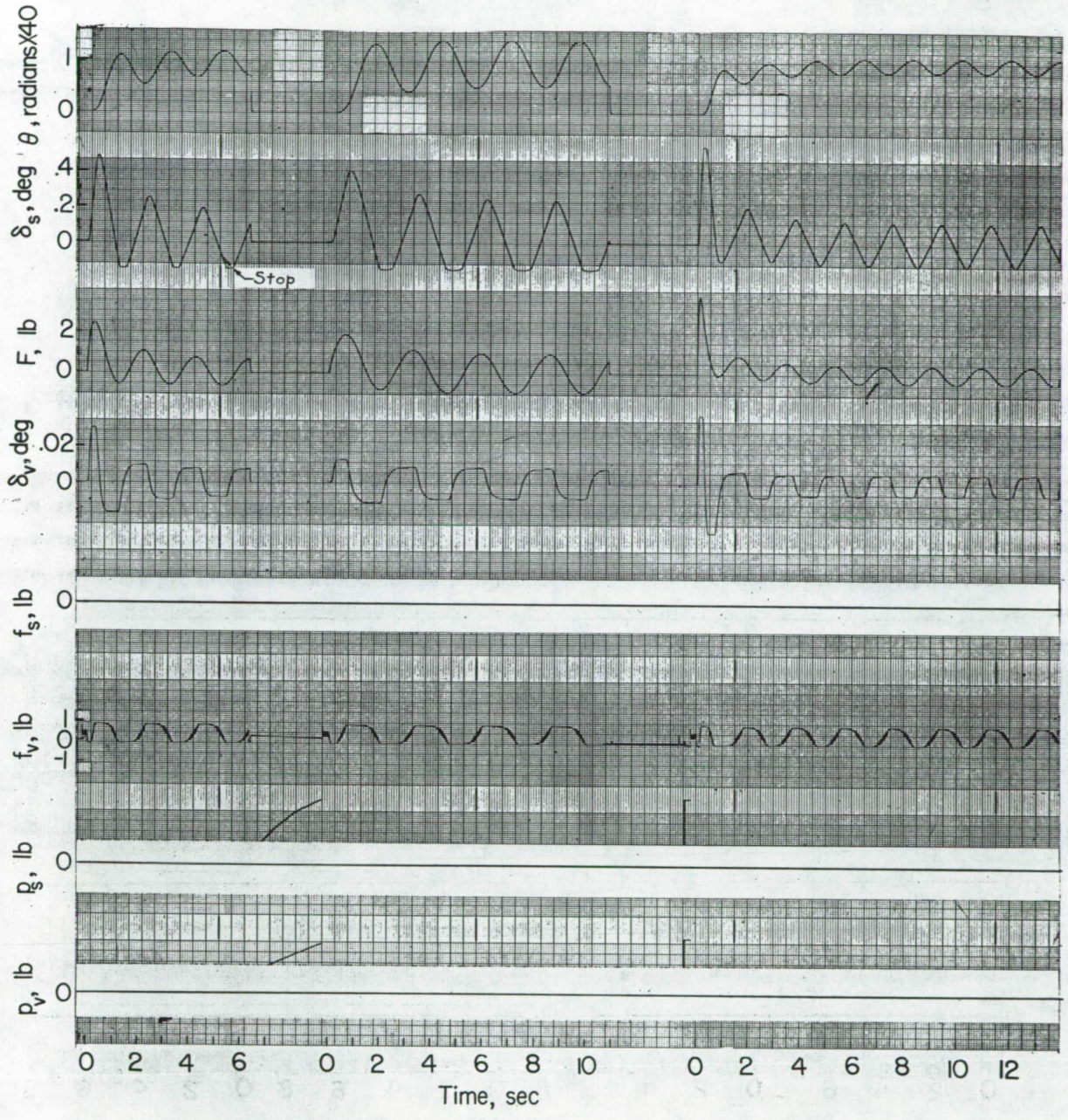
Figure 20.- Response time histories with both frictions and both preloads present for two values of desired attitude change. f_S = 1/2 pound; f_V = 1/2 pound; p_S = 1 pound; p_V = 1 pound.



(a) f_S = 1/2 pound;
f_V = 1/8 pound; K_b = 0.1.

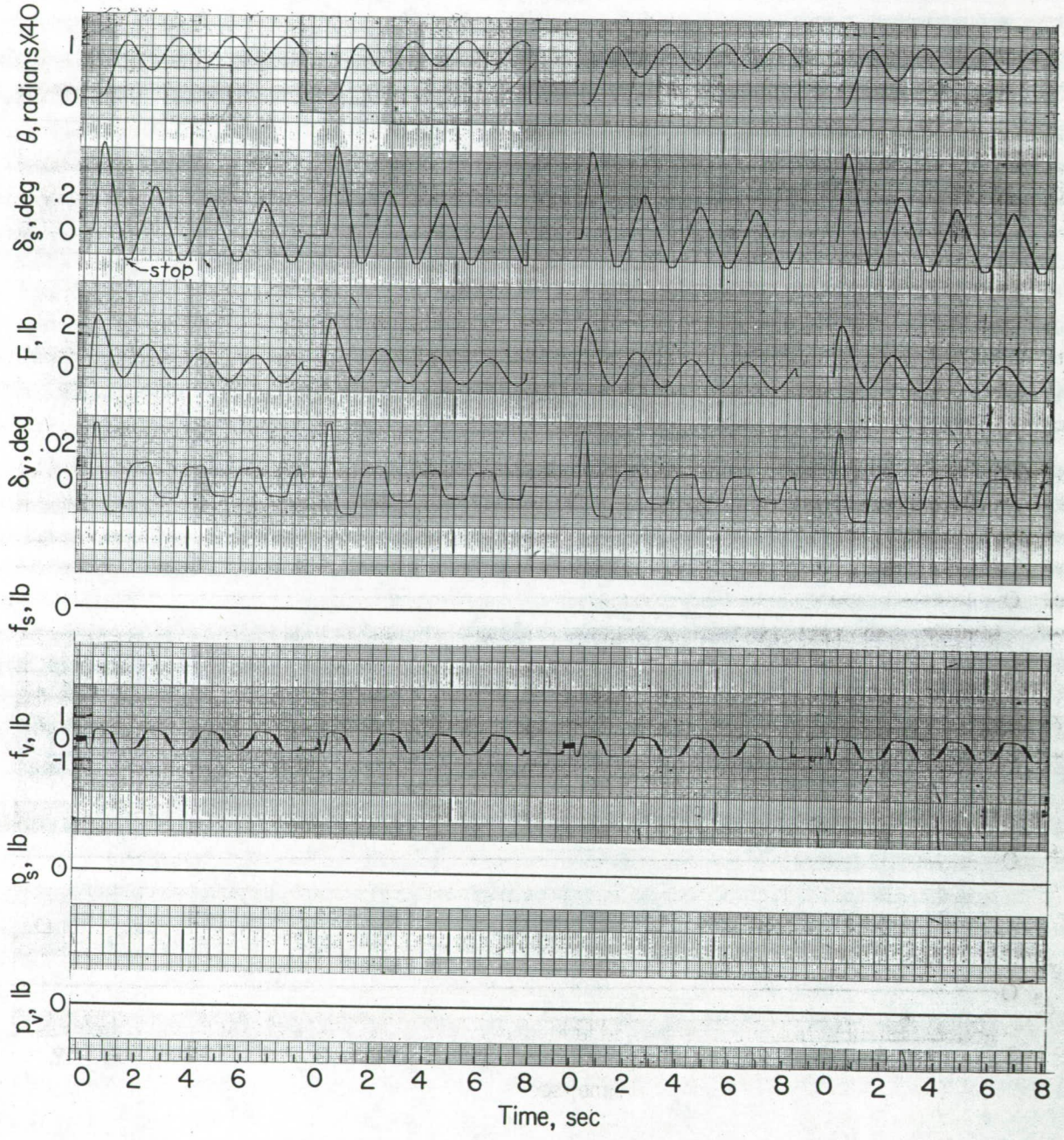
(b) f_S = 1/2 pound;
f_V = 1 pound; K_b = 0.8.

Figure 21.- Time histories which illustrate the variation in the effect of a given amount of valve friction on the closed-loop response when the error gearing K_b is varied. (Note that all force values stated are those felt at the stick.)



(a) τ_p = 0.15 sec. (b) τ_p = 0.30 sec. (c) τ_p = 0.075 sec.

Figure 22.- Time histories to illustrate the effect of variation in pilot lag on hunting oscillations of the closed-loop system induced by valve friction (1/2 lb).



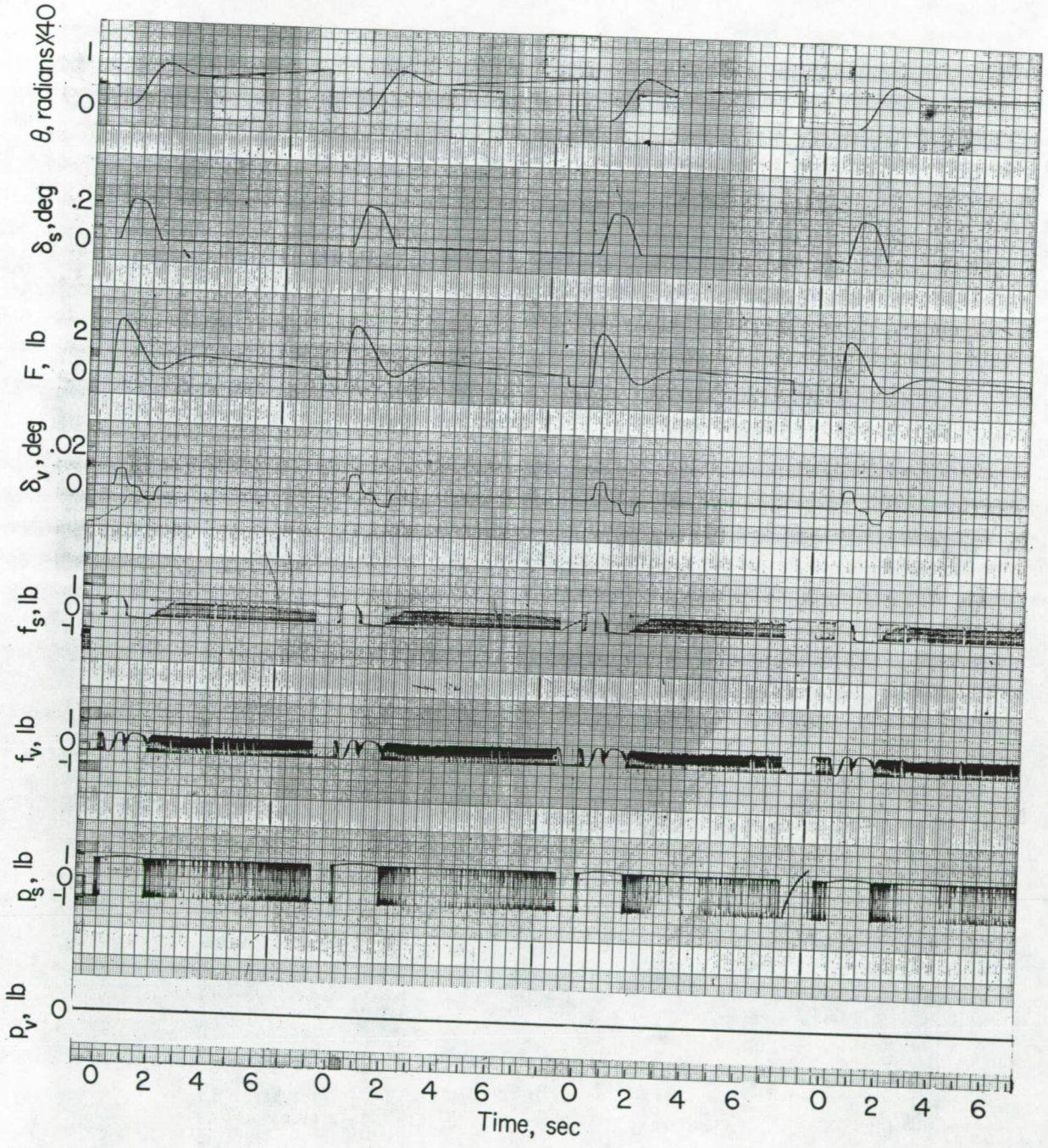
(a) $K_V = 0$; $C_V = 100$.

(b) $K_V = 0$; $C_V = 0$.

(c) $K_V = 573$; $C_V = 0$.

(d) $K_V = 573$; $C_V = 100$.

Figure 23.- Time histories to illustrate the negligible effect of eliminating valve centering and damping when the closed-loop system had neutral dynamic stability. K-units are ft-lb/radian; C-units are ft-lb/radian/sec.



(a) $K_V = 573$; $C_V = 100$.

(b) $K_V = 573$; $C_V = 0$.

(c) $K_V = 0$; $C_V = 0$.

(d) $K_V = 0$; $C_V = 100$.

Figure 24.- Time histories to illustrate the negligible effect of eliminating the valve centering and damping when the closed-loop system was stable.

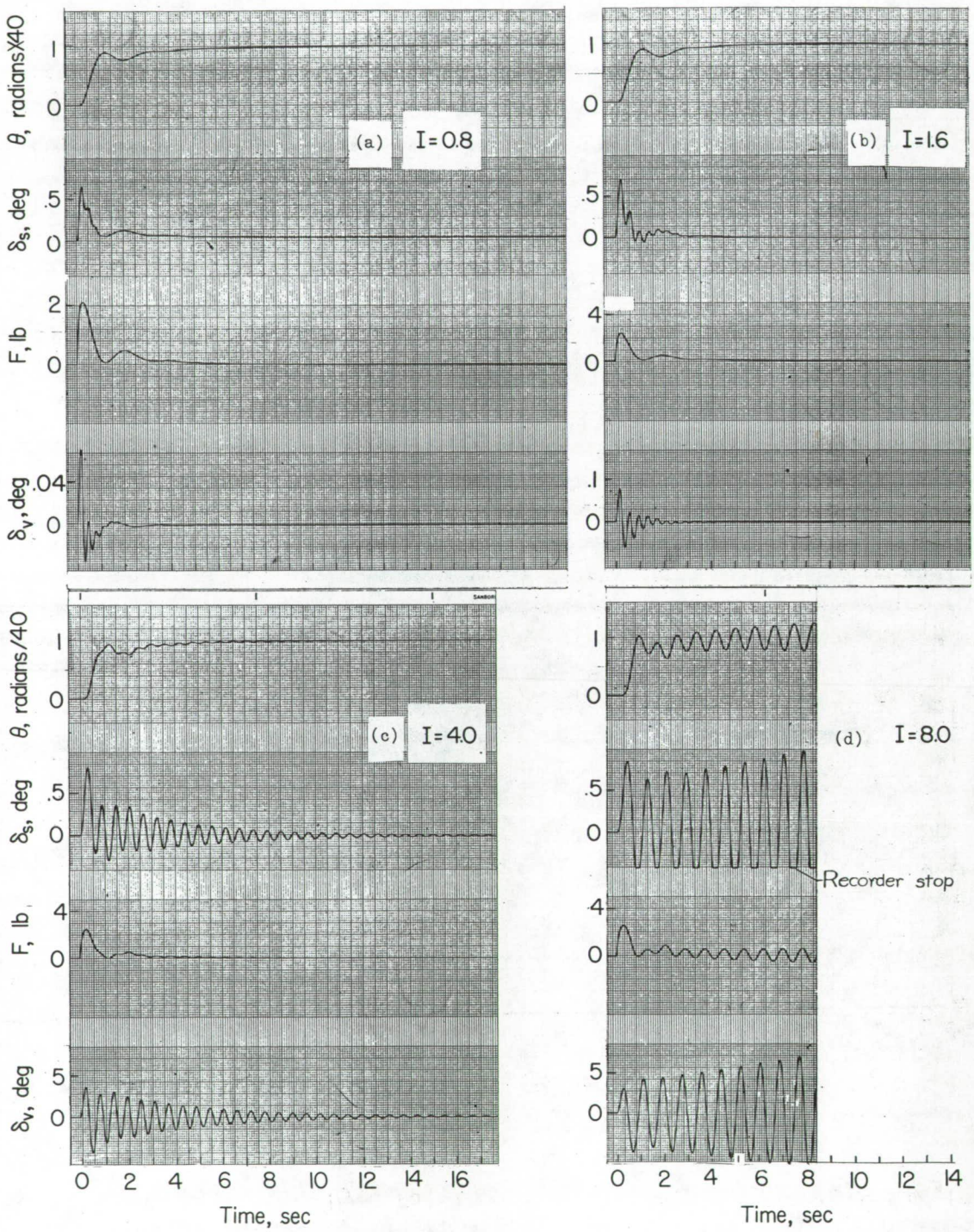
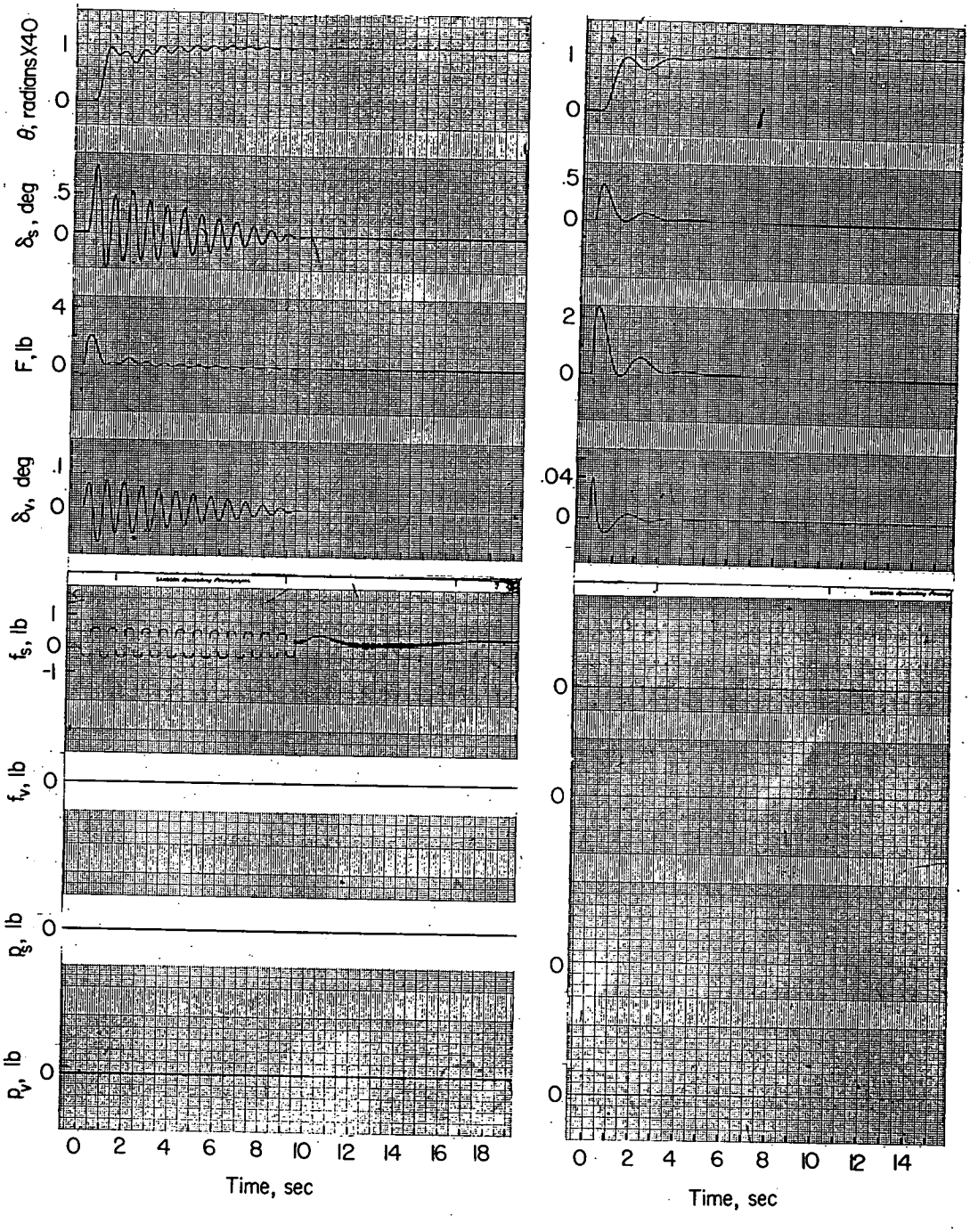


Figure 25.- Time histories which illustrate the effects of progressive increases in control-system inertia when the control system had no friction and no source of damping.



(a) f_S = 1/2 pound; C_S = 0.

(b) f_S = 0; C_S = 44.8 ft-lb/radian/sec.

Figure 26.- Time histories which illustrate the use of either a moderate amount of stick friction or the standard value of stick damping to stabilize the high inertia system.

CLASSIFICATION cancelled

CLASSIFICATION cancelled